Backaction Noise in Strongly Interacting Systems: The dc SQUID and the Interacting Quantum Point Contact

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We study the backaction noise and measurement the efficiency (i.e., noise temperature) of a dc SQUID amplifier and, equivalently, a quantum point-contact detector formed in a Luttinger liquid. Using a mapping to a dissipative tight-binding model, we show that these systems are able to reach the quantum limit even in regimes where several independent transport processes contribute to the current. We suggest how this is related to the underlying integrability of these systems.

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There has been considerable recent interest in studying detectors and amplifiers which add the minimal possible noise allowed by quantum mechanics to an input signal [1–7]. Such detectors are necessary for single spin and gravity wave detection, as well as for quantum control and quantum computation. In the important case where the detector is a mesoscopic conductor, understanding whether one can achieve the quantum limit (i.e., have a minimal backaction effect) requires an understanding of both the output current noise of the system as well as its backaction charge noise. Theoretically, these quantities have been studied for a variety of mesoscopic detectors, including noninteracting tunneling point-contact detectors [1,3] and more general coherent scattering detectors [6,7].

Here, we examine the ideality of mesoscopic detectors where nontrivial correlation effects are important. One such system is a quantum point-contact (QPC) detector formed in an interacting Luttinger liquid; this could be realized in a quantum Hall edge state or by embedding a noninteracting QPC in a resistive electromagnetic environment [8]. QPCs are in widespread use as readouts of quantum-dot qubits; in the absence of interactions, they are known to be able to reach the quantum limit [1,5–7]. What happens now when interparticle interactions are turned on? Note that while the current noise of an interacting QPC has received considerable attention [9,10], its backaction noise has not been studied.

A second correlated detector of obvious practical importance is the dc SQUID amplifier. While experimentally it is known that the dc SQUID can approach near quantum-limited operation, the theoretical limit for this system has not been fully studied. Previous studies either neglected the effect of the SQUID inductance [4] or were based on the quantum Langevin equation [11,12], an equation which is formally valid only in the limit of high temperatures or extreme dissipation [13].

In this Letter, we calculate the backaction noise and measurement efficiency (i.e., noise temperature) of both the interacting QPC detector and the dc SQUID using controlled perturbative approaches. We discuss how in each case the principle of detection is the same: the input signal modulates the tunneling of excitations across the detector. In the QPC, these excitations are electrons or quasiparticles; in the SQUID, they are Cooper pairs. We address both the weak-tunneling and strong-tunneling regimes of these systems. In the latter regime a multitude of tunneling processes, each transferring a different number of particles, will be significant. As the output of the detector essentially averages over these processes, one would expect there to be lost information and hence excess backaction, much in the same way that multiple channels in a noninteracting QPC lead to a departure from the quantum limit (QL) [6,7]. We show that this is not the case: one can remain at the QL even when multiple tunnel processes contribute. We suggest how this result is related to the integrability of the underlying field theory.

Models.—We start with the dc SQUID. Figure 1 shows a typical setup: two identical Josephson junctions, each shunted by a resistance R and capacitance C, placed in a ring of inductance L threading an external flux Φ_x. The junctions are symmetrically biased by a current bias I_B. We consider the typical case of nonhysteretic operation, and where $I_B > 2I_C$ ($I_C = 2eE_J/\hbar$ is the critical current of each junction). The dc SQUID is a flux-to-voltage amplifier. The input signal is a small additional flux $\Phi_{\rm in}$ which adds to Φ_x; by varying its value, one changes the voltage V across the SQUID. The backaction here is created by the circulating current J around the SQUID loop [11]. J directly couples to the input flux; its fluctuations act as a noisy backaction force on the input system.

We consider the dc SQUID as a linear detector, which requires only that the signal flux $\Phi_{\rm in}$ be small enough that the corresponding change in V is $\propto \Phi_{\rm in}$. A measure of the ideality of any linear amplifier or detector is its noise

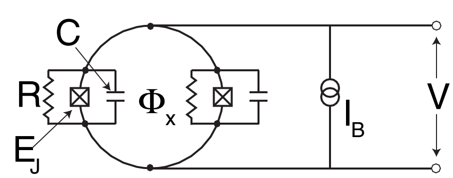

FIG. 1. Schematic of a dc SQUID amplifier.

temperature $k_B T_N$. It quantifies the total noise added to the input by the amplifier. Quantum mechanics requires a certain minimal amount of backaction; as a result, there is a quantum limit on T_N: $k_B T_N \geq \hbar\omega/2$, where ω is the signal frequency [14]. For an optimal coupling between the signal source and the SQUID, T_N is set by the noise of the uncoupled detector [15]:

$$\chi \equiv \frac{k_B T_N}{\hbar\omega/2} = \sqrt{\frac{S_V(\omega)S_J(\omega) - [\mathrm{Re}S_{VJ}(\omega)]^2}{[\hbar\lambda(\omega)/2]^2}} \geq 1 \quad (1)$$

Here, S_V (S_J) is the symmetrized noise in the output (backaction force) of the detector and S_{VJ} is the cross-correlation noise. $\lambda = d\langle V\rangle/d\Phi_x$ is the gain. For a dc SQUID with identical junctions, $S_{VJ} = 0$. We will focus on the low-ω limit of T_N; it describes the typical situation where ω is much smaller than detector frequencies. If our detector is used for qubit detection, the same limit of $1/\chi^2$ yields the measurement efficiency ratio [6,7].

We now analyze the SQUID. Heuristically, the bias current I_B will partially flow through the shunt resistors, and partially through the junctions. The junction current will be due to the incoherent tunneling of Cooper pairs [16]: Cooper pairs tunnel through the junction, simultaneously dissipating energy in the electromagnetic environment formed by the shunts. The voltage across the SQUID will then be set by the current flowing through the shunts: $V = (R/2)[I_B - (I_{\mathrm{CPT},1} + I_{\mathrm{CPT},2})]$, where $I_{\mathrm{CPT},j}$, the Cooper-pair tunneling current through junction j ($j = 1, 2$), depends both on Φ_x and I_B. Both the gain λ and output noise S_V can be related to I_{CPT}; the backaction circulating current will be given by $J = (I_{\mathrm{CPT},1} - I_{\mathrm{CPT},2})/2$.

The above picture is made rigorous by using a Caldeira-Legget representation of the impedances and a path-integral representation of the Keldysh partition function $\mathcal{Z}$ for the system [16]. Tracing out the environment, one has $\mathcal{Z} = \int \mathcal{D}\theta\mathcal{D}\phi \exp(\frac{i}{\hbar}\int dt \mathcal{L}[\theta, \phi])$. Setting $\phi = (\delta_1 + \delta_2)/2$ and $\theta = (\delta_1 - \delta_2)/2 - \pi\Phi_x/\Phi_0$, and using the indices c (q) to denote classical (quantum) Keldysh fields [17] ($\phi_\pm = \phi_c \pm \phi_q$, $\theta_\pm = \theta_c \pm \theta_q$), we have

$$\frac{\mathcal{L}}{2} = \sum_{\sigma=\pm}\left[\sigma E_J \cos\left(\theta_\sigma(t) + \frac{\pi\Phi_x}{\Phi_0}\right)\cos[\phi_\sigma(t) + vt]\right] + \sum_{\alpha=\phi,\theta}\int dt' \begin{pmatrix}\alpha_c(t)\\ \alpha_q(t)\end{pmatrix} \hat{G}_\alpha^{-1}(t-t') \begin{pmatrix}\alpha_c(t')\\ \alpha_q(t')\end{pmatrix}. \quad (2)$$

The effective voltage bias $V_B = RI_B$ sets $v = 2eV_B/\hbar$. The matrix Green functions $\hat{G}_\alpha$ ($\alpha = \theta, \phi$) are given in terms of the corresponding impedances Z_α via ($\hbar = 1$)

$$\frac{\omega\hat{G}_\alpha(\omega)}{4e^2} = \begin{pmatrix} -2i\mathrm{Re}Z_\alpha(\omega)\coth(\frac{\omega}{2k_BT}) & -iZ_\alpha(\omega) \\ iZ_\alpha^*(\omega) & 0 \end{pmatrix}, \quad (3)$$

with $Z_\phi(\omega) = (2/R + 2i\omega C)^{-1}$ and $Z_\theta(\omega) = [2/R + 2i\omega C + 2/(i\omega L)]$.

Introducing sources to calculate V, J, and their fluctuations from $\mathcal{Z}$, one can identify operators for the total Cooper-pair current I_{CPT} and the backaction current J. They take the expected form

$$I_{\mathrm{CPT}} = 2I_C \cos(\theta + \pi\Phi_x/\Phi_0)\sin(\phi + vt), \quad (4)$$

$$I_{\mathrm{diff}} = I_C \sin(\theta + \pi\Phi_x/\Phi_0)\cos(\phi + vt), \quad (5)$$

One now finds *rigorously* that $\langle V\rangle = (R/2)(I_B - \langle I_{\mathrm{CPT}}\rangle)$; moreover, at $T, \omega = 0$, one can directly relate the noise correlators of interest to the Cooper-pair shot noise: $S_V = (R/2)^2 S_{I_{\mathrm{CPT}}}$ and $S_J = S_{I_{\mathrm{diff}}}$.

At this point, we pause to note the analogy between the dc SQUID and the interacting QPC detector. The latter is a potential-to-current amplifier: the input signal modulates the tunneling amplitude Δ between two semi-infinite, spinless Luttinger liquid leads; the result is a corresponding modulation of the QPC current [18]. After bosonizing, the tunnel Hamiltonian is $\mathcal{H}_t = -\Delta\cos\phi$, and the Keldysh partition function for this system is identical to that for the dc SQUID *if* the phase θ is pinned. To make this correspondence [19], we associate the dimensionless conductance $1/\rho = h/(2e^2R)$ with the Luttinger interaction parameter g, $2E_J\cos\pi\Phi_x/\Phi_0$ with the QPC tunneling amplitude Δ, and $2V_B$ with the QPC bias voltage. The current through the QPC will then correspond to I_{CPT}, and the backaction force corresponds to $I_{\mathrm{diff}}/(I_C\sin\pi\Phi_x/\Phi_0)$. This mapping also requires Z_ϕ be constant over frequencies of interest, implying that the cutoff frequency $\omega_C = 1/(RC)$ be much larger than $v, eRI_C/\hbar$. We can thus view the dc SQUID as an interacting QPC detector where, due to fluctuations in θ, the magnitude of the tunnel matrix element is fluctuating.

We now return to the SQUID and address its backaction. By expanding $\mathcal{Z}$ in powers of E_J, we can describe processes involving multiple transfers of Cooper pairs. Such an expansion is well controlled in the large-I_B limit; for the experimentally relevant limit $\rho \ll 1$, the expansion converges for all $I_B > 2I_C$ [16]. The simplest limit is when only single Cooper-pair tunneling events play a role; for $\rho < 1$, this requires $I_C \ll I_B$. In this regime, it is sufficient to calculate to lowest nonvanishing order in E_J [4]. Introducing

$$\mathcal{J}_\alpha(t) = \frac{8e^2}{h}\int_0^\infty \frac{d\omega}{\omega}[\mathrm{Re}Z_\alpha(\omega)](e^{-i\omega t} - 1), \quad (6)$$

we find that the measurement efficiency (or reduced noise temperature) for an optimal $\Phi_x = \Phi_0/4$ is given by

$$\chi^2 = \left(e^{2\langle\theta^2\rangle}\frac{\int_0^\infty dt \sin(vt)\mathrm{Im}\{\exp[\mathcal{J}_\phi(t) + \mathcal{J}_\theta(t)]\}}{\int_0^\infty dt \sin(vt)\mathrm{Im}\{\exp[\mathcal{J}_\phi(t) - \mathcal{J}_\theta(t)]\}}\right)^2, \quad (7)$$

with $\langle\theta^2\rangle = (8e^2/h)\int_0^\infty d\omega\,\mathrm{Re}Z_\theta(\omega)/\omega$. We see that fluctuations in the phase θ prevent one from reaching the QL (i.e., $\chi = 1$). These fluctuations represent an extraneous source of noise: they reduce the gain of the detector and increase the backaction noise. Suppressing these fluctuations (i.e., pinning θ) requires that the loop inductance L be small. More precisely, for $\rho \ll 1$, Eq. (7) becomes

$$\chi^2 = \left(\frac{\mathrm{Re}Z_\phi(v) + \mathrm{Re}Z_\theta(v)}{\mathrm{Re}Z_\phi(v) - \mathrm{Re}Z_\theta(v)}\right)^2. \quad (8)$$

Reaching the QL in this large bias, weak-tunneling regime thus requires that Z_θ/Z_ϕ is small at the characteristic Josephson frequency v set by the bias current. Thus, in the $L \to 0$ limit, our system always reaches the QL as long as tunneling is weak, irrespective of further details of the impedance Z_ϕ. As the SQUID is equivalent to a interacting QPC in this limit, we can also conclude that *an interacting QPC in the weak-tunneling regime always reaches the QL, irrespective of* g. For repulsive interactions ($g < 1$), this regime corresponds to small voltages [18]. Using the usual duality between large and small g [19], this conclusion also holds for a QPC detector near perfect transmission, where the input signal modulates the strength of a weakly backscattering impurity. If the backscattering is weak enough that it can be treated to leading order (which requires large voltage for $g < 1$), one will again always be at the QL.

We next consider the $L \to 0$ limit, but now consider regimes where higher-order tunneling plays a role. Integrating out the phase ϕ in each term of $\mathcal{Z}$, we obtain a "Coloumb gas" description of $\mathcal{Z}$ [19]. We introduce auxiliary source fields in the action which couple to the Cooper-pair current and to the backaction force:

$$\mathcal{L}_{\mathrm{src}} = \sum_{\sigma=\pm} \sigma\{\eta(t) I_{\mathrm{CPT}}[\phi_\sigma(t)] + \lambda(t) J[\phi_\sigma(t)]\}. \quad (9)$$

To interpret the resulting expansion of $\mathcal{Z}$, we make an analogy to the Schmid model, a dissipative tight-binding (TB) model [20]. It describes a particle on a 1D lattice which feels a force from an oscillator bath having a spectral density $A(\omega) = (8e^2/h)\omega Z_\phi(\omega)$. In this mapping, $\tilde{E}_J = 2E_J \cos\pi\frac{\Phi_x}{\Phi_0}$ corresponds to the tunnel matrix element of the TB model, and $2eV_B$ to a constant external force. The expression for $\mathcal{Z}$ (at $\eta, \lambda = 0$) may now be cast as a sum over tunnel events which take the particle from an initial density matrix localized at $x = 0$ to one localized at $x = n$, where n is arbitrary. Each term in the expansion describes the amplitude of a process involving $2M$ tunnel events occurring at times t_1 to t_{2M}. Each event can move the particle either to the left or to the right, and can occur either on the forward or backwards Keldysh contour. Each tunnel event is thus labeled by a charge $\sigma_j = \pm 1$ which gives the direction of the tunnel event, and a charge ξ_j which determines the contour of the event: $\xi_j \sigma_j = 1\ (-1)$ for an event on the forward (backwards) contour. Finally, because of decoherence from the bath, each tunnel process results in a final density matrix state which is diagonal; we thus have $\sum \xi_j = 0$ for each term. We thus have $\mathcal{Z} = \lim_{t\to\infty} \sum_n P(n, t)$, with

$$P(n,t) = \sum_{M=n}^{\infty} \left(\frac{i\tilde{E}_J}{\hbar}\right)^{2M} \int_{-\infty}^{t} dt_1 \ldots \int_{t_{2M-1}}^{t} dt_{2M} \times \sum_{\vec{\xi},\vec{\sigma}} \left(\prod_{j=1}^{2M} e^{iv\xi_j t_j}[1 + \sigma_j \xi_j \lambda(t_j)] e^{i\eta(t_j)\sigma_j} \right) \times F[\vec{\sigma}, \vec{\xi}, \vec{\tau}]. \quad (10)$$

n is the net displacement of a given process, and the factor F describes interactions among the charges arising from integrating out ϕ; its precise form is given in Refs. [16,19,21]. Sums over charges in Eq. (10) are restricted to those satisfying $\sum_j \xi_j = 0$ and $\sum_j \sigma_j = 2n$.

We see that the source field η couples to the time derivative of the net displacement $n = \sum \sigma_j/2$. Thus, the Cooper-pair current I_{CPT} corresponds to the velocity of the particle in the TB model. This has been used previously to calculate the current noise in this system [22]. We also see that the source λ couples to $\sigma_j \xi_j$. Thus, the backaction force corresponds to the time derivative of the "quantum charge" $z = \sum \sigma_j \xi_j/2$; z is the net number of forward contour minus backward contour tunnel events. For a given M and n, z may range from $-(M - |n|)$ to $M - |n|$.

To make further progress, we follow Refs. [21,22] and consider the Laplace transform representation of $\mathcal{Z}$. As detailed in Refs. [21,22], one may then discuss $\mathcal{Z}$ in terms of "irreducible clusters." Letting s denote the Laplace transform variable, these are tunnel processes (i.e., a set of ξ and σ charges) which, in the $s \to 0$ limit, yield a finite constant contribution to $s[d\mathcal{Z}/dt](s)$. Unlike Refs. [21,22], we track both the classical displacement n and the quantum charge z; each irreducible process is characterized by a particular value of n and z. In the long time limit, each such process is statistically independent, with its amplitude $\gamma_{n,z}$ acting as an independent rate; one obtains Poissonian statistics for both n and z. At $T = 0$ and finite bias, one finds that all rates with $n < 0$ vanish. One can also show $\gamma_{n,-z} = (\gamma_{n,z})^*$. A straightforward calculation then yields

$$\frac{\langle I_{\mathrm{CPT}}\rangle}{2e} = \sum_{n=0}^{\infty} n\left(\sum_z \gamma_{n,z}\right) \equiv \sum_{n=0}^{\infty} n\Gamma_{cl}(n), \quad (11)$$

$$\frac{S_{I_{\mathrm{CPT}}}}{4e^2} = \sum_{n=0}^{\infty} n^2\left(\sum_z \gamma_{n,z}\right) \equiv \sum_{n=0}^{\infty} n^2\Gamma_{cl}(n), \quad (12)$$

$$\frac{S_J}{e^2 B} = \sum_{z=-\infty}^{\infty} z^2\left(\sum_{n=0}^{\infty} \gamma_{n,z}\right) \equiv \sum_z z^2\Gamma_q(z), \quad (13)$$

where $B = -\tan^2(\pi\Phi_x/\Phi_0)$. We can interpret $\Gamma_{cl}(n) = \sum_z \gamma_{n,z}$ as a "rate" associated with the incoherent tunneling of n Cooper pairs; each such process contributes to the current independently. Moreover, we see that backaction noise results from an uncertainty in how tunnel events are distributed between the two Keldysh contours (i.e., z fluctuates).

As many independent processes contribute to the current, one's first guess is that the system will no longer be at the QL (i.e., $\chi > 1$), as generically, there is lost information associated with averaging over the different transport processes. To address this issue, we first note a remarkable result found by Saleur and Weiss: the rates $\Gamma_{cl}(n) = \sum_z \gamma_{n,z}$ are *exactly* proportional to E_J^{2n} and contain no higher-order terms [22]. Formally, this relation results from cancellations of terms arising in perturbation theory. It implies $\Gamma_{cl}(n) = \gamma_{n,0}$, and that irreducible processes involving *both* backwards and forward tunneling events make no net contribution to charge transport.

Using the above result, along with $\lambda = \frac{d\langle I_{\rm CPT}\rangle}{d\Phi} = -\tan(\pi\Phi_x/\Phi_0)\frac{\pi\tilde{E}_J}{\Phi_0}\frac{d\langle I_{\rm CPT}\rangle}{d\tilde{E}_J}$, we see that a sufficient condition for having $\chi = 1$ is $\Gamma_{cl}(n) = -2\,\mathrm{Re}\Gamma_q(z = n)$. We have explicitly calculated the rates $\gamma_{n,z}$ to order $(E_J)^6$, which involves calculating 10 partial rates $\gamma_{n,z}$, each of which has contributions from numerous charge sequences. We find that a much stronger relation is satisfied: if *both* n and z are nonzero,

$$\mathrm{Re}\,\gamma_{n,z} = 0, \tag{14}$$

otherwise,

$$-2\,\mathrm{Re}\gamma_{n=0,z} = \gamma_{n=z,0} \propto (E_J)^{2z}. \tag{15}$$

Note the duality between transport in the classical (i.e., n) and quantum (i.e., z) directions. The only irreducible process yielding a classical displacement n has precisely $2n$ tunnel events all occurring in the same direction. Similarly, the only irreducible process increasing the quantum charge by Δz has exactly $2(\Delta z)$ tunnel events on the same Keldysh contour. The cancellations which yield these results, and thus a quantum-limited backaction, require a purely Ohmic bath spectrum. This is always the case for the interacting QPC. For the dc SQUID, one needs the Josephson frequency v to be much smaller than the cutoff frequency $1/(RC)$ in order to reach the quantum limit at strong tunneling. Note that Eqs. (14) and (15) *are not* a direct consequence of the result for $\Gamma_{cl}(n)$ found in Ref. [22]. Also note that, using duality [16,19], our conclusions also apply to the case where a perturbative expansion in E_J does not converge, as one can similarly analyze an alternate convergent expansion for $\mathcal{Z}$.

The remarkable cancellations that lead to the result of Eqs. (14) and (15) are, similar to the result $\Gamma_{cl}(n) \propto (E_J)^{2n}$, a result of the integrability of the model studied here. Integrability may be used to exactly calculate the current and current noise in this system [10]. The solution is given in terms of the scattering of quasiparticles in a boundary sine-Gordon model. One calculates an energy-dependent quasiparticle transmission coefficient $\mathcal{T}(\varepsilon)$, and then uses this to calculate the current and noise. Note that for a *noninteracting* QPC, where again the signal of interest modulates a localized potential, reaching the QL requires the QPC transmission $\mathcal{T}(\varepsilon)$ to satisfy [6,7]

$$\frac{dT(\varepsilon)}{d\Delta} = CT(\varepsilon)[1 - T(\varepsilon)]. \tag{16}$$

Here Δ is the strength of the QPC backscattering potential, and C is an energy-independent constant. Remarkably, the energy-dependent transmission $\mathcal{T}(\varepsilon)$ for quasiparticles in the boundary sine-Gordon model *satisfies the exact same equation* [see Eq. (13) of Ref. [10]]. This suggests a deep connection between integrability and the QL. Note also that by using the results of Refs. [8,10], one can see that in a *weakly* interacting QPC, the "physical" transmission coefficient (i.e., for electrons, not for quasiparticles) satisfies Eq. (16); this follows from the fact that in the exact solution of Ref. [10], $d\mathcal{T}/d\Delta \propto d\mathcal{T}/d\log(E)$, where E is energy. Thus, if one is concerned with the validity of Eq. (16) on the physical transmission coefficient, this is also known to be satisfied in a weakly interacting QPC.

In conclusion, we have calculated the backaction noise and measurement efficiency for both the dc SQUID amplifier and the interacting QPC detector. Using a perturbative approach, we have shown that it is still possible to reach the quantum limit in regimes where multiple higher-order tunneling processes play a role.

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