Interaction-Induced Restoration of Phase Coherence

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We study the conductance of a quantum "T junction" coupled to two electron reservoirs and a quantum dot. In the absence of electron-electron interactions, the conductance g is sensitive to interference between trajectories which enter the dot and those which bypass it. We show that including an intradot charging interaction has a marked influence: it can enforce a coherent response from the dot at temperatures much larger than the single-particle level spacing Δ. The result is large oscillations of g as a function of the voltage applied to a gate that is capacitively coupled to the dot. Without interactions, the conductance has only a weak interference signature when $T > \Delta$.

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How do interactions affect the phase coherence of electrons traveling through a quantum dot? This question is of interest both because of the fundamental issues it raises and because of its relevance to recent experiments. Groundbreaking experiments in which a Coulomb blockaded quantum dot is embedded in an Aharonov-Bohm ring [1] have demonstrated that transport through such dots is at least partially coherent, despite strong interactions. Complementary studies which observed Fano resonances in the conductance of such dots also substantiate this conclusion [2], while the observation of weak localization and conductance fluctuations has demonstrated coherence in open (i.e., not Coulomb blockaded) dots [3]. In general, electron-electron interactions are expected to degrade the coherence of transport through a dot—interference phenomena, such as the amplitude of Aharonov-Bohm oscillations in the conductance are suppressed compared to the noninteracting case [4]. In this Letter we study a system in which the opposite phenomenon occurs: the presence of a charging interaction in a quantum dot significantly enhances interference phenomena compared to the situation without interactions [5].

Motivated by the experiments mentioned above, we consider a T-junction system which is sensitive to the coherence of electrons *reflected* from a quantum dot. The T junction consists of three coincident single-mode quantum point contacts coupled to source and drain reservoirs and to a quantum dot (Fig. 1). Without interactions, the source-drain conductance g is sensitive to constructive or destructive interference between trajectories which bypass the dot, and those which enter it. If electron motion in the dot is fully coherent, g (in units of e^2/h) is [6]

$$g = g_{\max} \int d\varepsilon \left(-\frac{df}{d\varepsilon}\right) \sin^2[\delta + \alpha(\varepsilon)]. \qquad (1)$$

Here δ is the transmission phase shift associated with direct trajectories bypassing the dot (Fig. 1a), $\alpha(\varepsilon)$ is the phase shift for scattering from the dot through an effective tunnel junction with reflection probability $|r|^2$ (Fig. 1b) [7], and f is the Fermi function. The parameters δ, $|r|$, and $g_{\max}$ are determined by the 3×3 scattering matrix S of the T junction without dot [8]. This matrix changes appreciably only over energies comparable to the Fermi energy E_F, and can be treated as constant over the smaller scales we focus on. At zero temperature, the conductance (1) exhibits full constructive and destructive interference as a function of the phases δ and α, resulting in oscillations of g between 0 and its maximum value $g_{\max}$. While δ is a property of the T junction and cannot be tuned, α depends on the dot and can be varied, e.g., by changing E_F. The typical energy scale for variations of α is Δ, the single-particle level spacing of the dot.

In this Letter, we focus on the temperature regime $T \gg \Delta$. Here the signatures of interference in Eq. (1) are washed out by thermal smearing, resulting in $g = \frac{1}{2} g_{\max}(1 - |r| \cos 2\delta)$. We ask how this now changes when the effects of intradot electron-electron interactions are included. We consider the capacitive interaction

$$H_C = E_C(n_{\text{dot}} - \mathcal{N})^2, \qquad (2)$$

where $E_C \ll E_F$ is the charging energy, n_{dot} the electron number on the dot, and $\mathcal{N}$ the dimensionless voltage of a gate electrode capacitively coupled to the dot. Although we consider $T \gg \Delta$, we also require $T \ll E_C$, so that charged dot excitations are suppressed. Our main result is that in this regime transport through the T junction is more coherent with interactions than without—electrons scatter from the dot with a well defined $\mathcal{N}$-dependent phase, resulting in interference and hence an $\mathcal{N}$ dependent conductance. The origin of the resulting oscillations is entirely

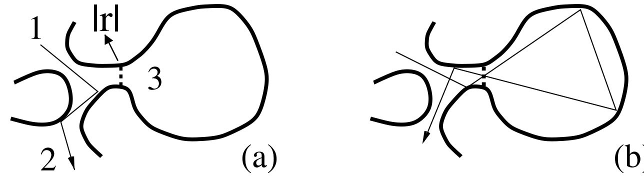

FIG. 1. (a) Schematic of the T-junction plus dot system, showing a direct trajectory; such scattering events are described by the phase δ. The effective tunnel junction in the entrance to the dot has a reflection probability $|r|^2$. (b) Same as (a), showing a scattering event which involves the dot.

different from that of standard Coulomb blockade oscillations, as there is no "blockade" here—electrons traveling from source to drain are not forced to pass through the dot. We find this result remarkable, as one usually expects that electron-electron interactions degrade, rather than enhance, coherence, due to the possibility of creating low-energy particle-hole excitations in the dot.

The underlying reason for this enhanced coherence is a subtle consequence of the charging interaction first discussed by Matveev [9]. Using the convention

$$\mathcal{N} = N_0 + 1/2 + x, \qquad |x| \leq 1/2, \tag{3}$$

where N_0 is an integer, for $T \ll E_C$ only the states $n_{\rm dot} = N_0$ and $n_{\rm dot} = N_0 + 1$ are dynamically significant. These states may be regarded as being the $\downarrow$ and $\uparrow$ states, respectively, of a fictitious impurity spin. If we also assign lead (dot) electrons a fictitious $\uparrow$ ($\downarrow$) spin, then the tunneling Hamiltonian between lead and dot takes the form of spin-flip scattering off an impurity. In this way, the Coulomb blockade problem can be mapped onto an anisotropic Kondo model; for spinless electrons [10] this is a single-channel Kondo (1CK) model, while with spin it is a two-channel Kondo (2CK) model. In this analogy, $|t|$ plays the role of the dimensionless exchange constant $J\rho_0$, and x the role of a local impurity magnetic field. The coherent reflection from the dot results from this effective Kondo physics, as we will now discuss.

For a quantitative description, we write the Hamiltonian of the T junction and dot as $H = H_D + H_L + H_S + H_C$, where $H_D = \sum_{\alpha\beta,\sigma} \mathcal{H}_{\alpha\beta} d^\dagger_{\alpha\sigma} d_{\beta\sigma}$ is the Hamiltonian of the closed dot, $H_L = \sum_{j=1,2}\sum_\sigma \int dk\, \varepsilon(k)\psi^\dagger_{j\sigma}(k)\psi_{j\sigma}(k)$ is the kinetic energy of electrons in leads 1 and 2, and H_C is given in Eq. (2). Scattering in the T junction is described by

$$\begin{aligned} H_S = \sum_{\sigma,i=1,2} \int dk \Bigg[\sum_{j=1}^{2} \int dk'\, W_{ij}\psi^\dagger_{i\sigma}(k)\psi_{j\sigma}(k') \\ + \sum_\alpha [W_{i3}\psi^\dagger_{i\sigma}(k) d_{\alpha\sigma} + \text{H.c.}] \Bigg] \\ + \sum_{\sigma,\alpha,\beta} W_{33} d^\dagger_{\beta\sigma} d_{\alpha\sigma}\,, \end{aligned} \tag{4}$$

where the 3×3 Hermitian matrix W describes a potential corresponding to the scattering matrix $S(E_F)$. The dot electron number $n_{\rm dot}$ in Eq. (2) reads $n_{\rm dot} = \sum d^\dagger_{\alpha\sigma} d_{\alpha\sigma}$.

We start with the case of a weakly coupled dot ($|t|^2 \ll 1$). Using an approach similar to that of Ref. [11], we express the source-drain conductance in terms of the single-particle retarded Green function $G^R_{\alpha\beta}(\omega)$ of the dot. As the dot is coupled to only a single point contact, and as G^R is diagonal in spin, one can obtain an exact expression involving *only* G^R evaluated at the contact [6]:

$$\frac{g}{g_{\max}} = \sin^2\delta + \frac{\Gamma}{4}\,\mathrm{Im} e^{2i\delta} \sum_\sigma \int d\varepsilon \frac{df}{d\epsilon} G^R_\sigma(\varepsilon)\,, \tag{5}$$

where $\Gamma = \frac{1}{2\pi}|t|^2\Delta$. In the regime of weak tunneling and $T > \Gamma$, it is possible to do a lowest order calculation in $|t|^2$ by using G_R for an uncoupled dot, which can be obtained exactly [12]. Averaging over fluctuations of the dot wave functions, and treating dot occupation factors in the same way as the rate-equations approach [13], we find to lowest order in $|t|^2$, when $\Delta \ll T \ll E_C$,

$$\begin{aligned} \frac{g}{g_{\max}} = \frac{1}{2} - \cos(2\delta)\left[\frac{1}{2} - \frac{|t|^2}{4}\frac{E_C x/T}{\sinh(2E_C x/T)}\right] \\ + \sin(2\delta)\left[\frac{|t|^2}{8} Y(x)\right]. \end{aligned} \tag{6}$$

Near resonance $|x| \ll 1$, $Y(x)$ is approximately

$$Y(x) = \frac{2}{\pi}\tanh(E_C x/T)\log\left[\min\frac{2E_C}{T}, \frac{1}{|x|}\right]. \tag{7}$$

The first $|t|^2$ correction to the conductance in Eq. (6) is proportional to $\mathrm{Im}G^R$, and is identical to the conductance through a Coulomb blockaded dot coupled to two leads when $\Delta \ll T \ll E_C$ [13]. This term is always small ($\sim|t|^2 \ll 1$). In contrast, the second correction term to the conductance, arising from $\mathrm{Re}G^R$, gives rise to a low-temperature logarithmic divergence near resonance. Its origin is a partial cancellation between electron-like processes ($n_{\rm dot} = N_0 \to N_0 + 1$) and holelike processes ($n_{\rm dot} = N_0 + 1 \to N_0$), which is identical to how a logarithmic divergence arises in the conventional Kondo problem. This is not surprising, given the analogy already discussed. The logarithm in Eq. (7) is cut off by x, consistent with x playing the role of a magnetic field in the Kondo analogy.

To summarize, we see that the lowest order in $|t|$ conductance calculation indicates an instability which enhances the $\mathcal{N}$ dependence of g at low temperatures. As the only way an $\mathcal{N}$ dependence can arise in the T-junction geometry is via interference, this breakdown suggests an enhancement of coherent scattering from the dot. In terms of the Kondo analogy, the failure of perturbation theory results from the instability of the weak-coupling fixed point. The effective Kondo temperature which characterizes this instability is $T_K \sim E_C e^{-c/|t|}$ (with c a constant) [9], consistent with the fact that $|t|$ is analogous to a dimensionless exchange coupling.

To investigate the regime T, $E_C x \leq T_K$, where Eq. (6) fails and Kondo physics becomes dominant, we now present results of a calculation for the opposite situation of a strongly coupled quantum dot ($|t| \simeq 1$). For strong coupling, $T_K \to E_C$, and the condition $T < E_C$ ensures that we will be in a regime dominated by Kondo physics for all values of $\mathcal{N}$. To deal with a strongly coupled dot at $\Delta \ll T \ll E_C$, we use the approach of Flensberg [14] and Matveev [15–17]. In this approach the limit $\Delta \to 0$ is taken, and electron dynamics near the T junction is described using a one-dimensional model for each point contact. The interaction is treated exactly using bosonization, while the effects of weak backscattering ($|r| \ll 1$) are dealt with perturbatively [6]. In what follows, we discuss the case of spinless electrons and electrons with spin separately.

Without spin [10], our system corresponds to the single-channel Kondo model, which is well known to have a Fermi liquid (FL) ground state in which the magnetic impurity acts as a potential scatterer. We thus expect the $T < T_K \sim E_C$ properties of the open dot system to also conform to a Fermi liquid state. Indeed, a rigorous calculation gives to order $|r|^2$:

$$g = g_{\max} \sin^2(\delta + \pi n_{\rm dot}) + O(T/E_C)^2, \qquad (8)$$

where

$$n_{\rm dot} = \mathcal{N} - (e^C |r|/\pi) \sin 2\pi\mathcal{N} + \lambda_2 (e^C |r|/\pi)^2 \sin 4\pi\mathcal{N} + O(|r|)^3. \qquad (9)$$

Here C is Euler's constant, and $\lambda_2 \simeq 1.9$. Equation (8) indicates that despite being at $T \gg \Delta$, a regime where the noninteracting system is essentially incoherent, reflection from the interacting dot is fully coherent—as $\mathcal{N}$ is tuned, g exhibits full constructive and destructive interference. The scattering phase shift $\alpha = \pi n_{\rm dot}$ obeys the Friedel sum rule [18], as expected for the FL ground state of the single-channel Kondo model. Equation (8) confirms that in the spinless case the breakdown of perturbation theory in Eq. (6) indeed signals coherent scattering from the dot. The fact that a Fermi liquid picture holds in the spinless case (near $|r| = 0$) was first noted by Aleiner and Glazman [19].

A heuristic phase diagram describing the $\mathcal{N}$ dependence of the conductance for a fixed temperature $\Delta \ll T < E_C$ is given in Fig. 2a. For small $|r|$, $T < T_K$ for all $\mathcal{N}$, and the coherent expression of Eq. (8) holds for all $\mathcal{N}$ (i.e., one is always in region I). At $|r| \simeq 0$, $n_{\rm dot} \simeq \mathcal{N}$, and $g(\mathcal{N})$ has a sinusoidal form, while, for larger $|r|$, $n_{\rm dot}$ will change rapidly by 1 near resonance [15], implying a corresponding rapid change in the phase α by π, and hence a Fano-type line shape [20]. As we approach the weak-coupling regime ($|r| \to 1$), we will have $T_K < E_C$, meaning that this Kondo induced coherence will occur only at sufficiently low temperatures $T < T_K$ and close to resonance, $|x| < T_K/E_C$. We still expect a narrow Fano line shape in this regime, as all the interesting phase behavior occurs near resonance. For $T \geq T_K$, temperature cuts off scaling to the strong-coupling fixed point, and consequently there will be no enhancement of coherent scattering; the $\mathcal{N}$ dependence of g will remain weak, being described by Eq. (6).

Even though we are at $T \gg \Delta$, including spin changes the behavior of the stub considerably. The analogy is now to a two-channel Kondo model (the two spin projections act as the two conserved channels), which is markedly different from the one-channel case. At zero magnetic field (i.e., $x = 0$ in our system), the low-temperature properties of the 2CK model are described by a non-Fermi liquid (NFL) fixed point which corresponds to a dimensionless exchange constant of order unity (i.e., $|t| \simeq 1$). A nonzero

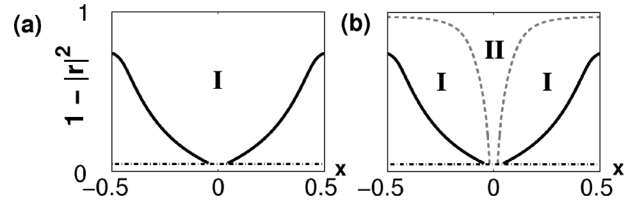

FIG. 2. Heuristic phase diagrams for the conductance g of the T junction, for a fixed $T \ll E_C$, without spin (a) and with spin (b). $|r|^2$ is the reflection probability from the dot entrance, and x is the dimensionless gate voltage ($x = 0$ implies charge degeneracy); cf. Eq. (3). In regions I and II, infrared Kondo fixed points determine the physics: the solid line indicates $E_C x = T_K(|r|)$, and the dashed line in (b) indicates $E_C x = \Gamma_c(|r|)$; see text. Scattering from the dot is mainly coherent in region I (implying g depends strongly on x here), whereas it is mainly incoherent in II and in the unlabeled region outside I. The dot-dashed line indicates $T = T_K(|r|)$.

magnetic field (i.e., $|x| > 0$) destroys the stability of this fixed point, and the system flows towards an alternate FL fixed point. Both these fixed points have an impact on the conductance, as we now demonstrate.

Perturbation theory in $|r|$ for $T \ll E_C \sim T_K$ yields the following form for the conductance:

$$g = \frac{g_{\max}}{2} [1 - \chi \cos(2\delta + 2\alpha)], \qquad (10)$$

where, defining $\Gamma_c = 2e^C \pi^{-2} |r|^2 E_C \sin^2(\pi x)$,

$$\chi(x, T) = c_1 \sqrt{\frac{\Gamma_c}{T}} + O\left(\frac{\Gamma_c}{T}\right)^{3/2}, \qquad (11)$$

$$\alpha(x, T) = \frac{\pi}{2}\left(\frac{1}{2} + x - \theta(x)\right) + O(|r|)^2. \qquad (12)$$

Here $c_1 \simeq 1.8$, and we have used Eq. (3) for the gate voltage $\mathcal{N}$; it follows that g is periodic in $\mathcal{N}$ with period 1. The order $|r|^2$ correction to α is proportional to $\sin 2\pi x$, and diverges only logarithmically at low T.

The first term of Eq. (10) can be interpreted as an incoherent contribution to the conductance, while the second term represents an interference contribution. At $|r| = 0$ only the former contributes, thus agreeing with what was found for the noninteracting system, but in stark contrast to the interacting spinless case; cf. Eq. (8). The complete incoherence at $|r| = 0$ corresponds to the vanishing probability for single-particle scattering at the NFL fixed point of the 2CK model [21]. Equivalently, one can think of fluctuations in the dot spin as suppressing a coherent response in this temperature regime [22]. For nonzero $|r|$, the second term in Eq. (10) also contributes. This term has an interference form, with a well-defined, x-dependent scattering phase α associated with the dot. The weight χ of this term is zero on resonance ($x = 0$), and grows at low temperatures when off-resonance ($x \neq 0$). These features can be understood within the 2CK analogy. A nonzero x makes the NFL fixed point unstable, and the resulting renormalization group flow at low temperatures is towards the FL fixed point, which has a well-defined phase shift for scattering from the dot [23]. This flow manifests itself here as a small coherent term in the conductance which grows at low temperatures. The flow is parametrized in Eq. (10) by the function $\chi(x, T)$; we expect $\chi \to 1$ in the vicinity of the FL fixed point [16].

Note that the scattering phase α in Eq. (12) is not simply proportional to $n_{\rm dot}$. (At $|r| = 0$, $n_{\rm dot} = \mathcal{N}$ [15].) Near the FL fixed point [i.e., $T \ll \Gamma_c(x)$], α can be obtained within the Kondo analogy using Fermi liquid arguments [24]. First, note that $(n_{\rm dot} - 1/2 - N_0)$ is equivalent to $\langle S_z \rangle$, the moment of the Kondo impurity spin. This moment will be equal to the bare moment of the impurity plus a quasiparticle contribution, which may be written in terms of phase shifts: $\langle S_z \rangle = \frac{1}{2}\mathrm{sgn}(x) + 2 \times \frac{1}{2}(\frac{\delta_\uparrow(x)}{\pi} - \frac{\delta_\downarrow(x)}{\pi})$. The factor of 2 corresponds to the two equivalent channels of the model. Finally, as the impurity spin has zero charge, one has $\delta_\uparrow = -\delta_\downarrow$. In the Kondo analogy, $\uparrow$ is associated with electrons in the lead. Equating α with $\delta_\uparrow$ then yields

$$\alpha = \frac{\pi}{2}[n_{\rm dot} - N_0 - \theta(x)] \qquad [\mathrm{mod}\, \pi], \tag{13}$$

which agrees with Eq. (12) [25].

A heuristic phase diagram is shown in Fig. 2b for a fixed temperature $\Delta \ll T \ll E_C$. For small $|r|$, $\Gamma_c(x) < T$ for all x, and one is always in region II: the incoherent term in Eq. (10) dominates, and $g(x)$ exhibits only weak oscillations. These will grow as $|r|$ is increased or as T is lowered, following Eq. (10). For sufficiently large $|r|$ (or low T), tuning x can take one from region II to region I, with the crossover occurring at $\Gamma_c(x) = T$ (dashed line in Fig. 2b). Near resonance (in II), g is still given by Eq. (10), but away from resonance (in I), it is given by the coherent expression $g = g_{\max} \sin^2(\delta + \alpha)$, where α is given by Eq. (13). We expect large oscillations in $g(\mathcal{N})$ in this regime, with a sharp feature emerging around $x = 0$ as T is lowered due to the rapid jump by $\pi/2$ in α.

The coherence effects discussed here are expected to be largely insensitive to additional sources of dephasing in the quantum dot (e.g., from external sources or from electron-electron interaction terms we neglect) if $|t|$ (and hence T_K) is sufficiently large. For strong coupling ($|t| \to 1$), the time an electron effectively spends in the dot before being reflected is $\sim\hbar/E_C$, which is much shorter than typical dephasing times [3]. Thus, electrons should still scatter coherently from the dot in this regime. Note that the model of Ref. [15] used for the strongly coupled dot already assumes that electron motion in the dot is completely incoherent.

Finally, our results may have relevance to the experiments in Ref. [1], as they indicate that the relation between $n_{\rm dot}$ and the scattering phase from an interacting dot may be significantly different than that expected for a noninteracting dot.

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[8] Using the labeling in Fig. 1a, one has $|r|^2 = |S_{33}|^2 \equiv 1 - |t|^2$, $g_{\max} = 8(|S_{31}S_{32}|^2)/|t|^4$, and $\delta = \arg(S_{31}S_{32})$. For spinless electrons, $g_{\max} = 4(|S_{31}S_{32}|^2)/|t|^4$.
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