# Shot noise of a tunnel junction displacement detector

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We study quantum-mechanically the frequency-dependent current noise of a tunnel-junction coupled to a nanomechanical oscillator. The cases of both dc and ac voltage bias are considered, as are the effects of intrinsic oscillator damping. The dynamics of the oscillator can lead to large signatures in the shot noise, even if the oscillator-tunnel junction coupling is too weak to yield an appreciable signature in the average current. Moreover, the modification of the shot noise by the oscillator cannot be fully explained by a simple classical picture of a fluctuating conductance.

Spurred primarily by experiments in solid-state qubit systems, there has recently been considerable interest in understanding the noise properties of mesoscopic systems used as detectors.[1–6] Many new results have emerged, including an understanding of the connection between noise, back-action dephasing and information,[4–6] and of the influence of coherent qubit oscillations on the output noise of a detector.[3] Not surprisingly, similar concerns arise in the study of nanomechanical oscillators. Recent experiments using single-electron transistors (SETs) have demonstrated displacement detection of such oscillators with a precision close to the maximum allowed by quantum mechanics.[7,8] Given the interest in these systems, it is important to gain a better understanding of how a mesoscopic detector influences the behavior of an oscillator, and vice-versa. Several works have addressed various aspects of this problem. In particular, it has been shown that an out-of-equilibrium detector can serve as an effective environment for the oscillator, providing both a damping coefficient and an effective temperature.[11–13]

In the present work, we study the finite-frequency output noise of a mesoscopic displacement detector, where one expects to see signatures of the time-dependent fluctuations of the oscillator. A completely classical study of the current noise of a SET displacement detector was presented recently in Refs. 14 and 15. In contrast, we consider a generic tunnel-junction or quantum point-contact (QPC) detector, in which the tunneling strength depends on the position of the oscillator, and calculate *quantum mechanically* the finite frequency current noise. Such a system could be realized by using a STM setup where one electrode is free to vibrate.[9,10] We treat both dc and ac voltage bias; the latter is of particular relevance to experiments.[10] We find that even for a detector-oscillator coupling so weak that there is little signature of the oscillator in the average current, there can nonetheless be a strong signature in the finite-frequency current noise. We moreover find that the oscillator contribution to the noise cannot be simply explained by a classical model of a detector conductance which fluctuates with the oscillator position—there are additional quantum corrections which suppress the contribution of zero point fluctuations. We show that these quantum corrections result from correlations between the detector's random back-action force and intrinsic output noise. Finally, in the ac-biased case, we find that the oscillator experiences a *time-dependent* temperature, which has a direct

influence on the detector's current noise. Note that the noise of a QPC position detector was also briefly considered in Ref. 16.

*Model.* Considering the simplest case where the tunnel-matrix element depends linearly on the oscillator displacement $\hat{x}$, the tunnel junction detector is described-by

$$H_{det} = \left( \frac{\tau_0 + e^{i\eta}\tau'\hat{x}}{2\pi\Lambda} \sum_{k,k'} Y^\dagger c^\dagger_{R,k} c_{L,k'} + \text{H.c.} \right) - eV(t)\hat{m}$$

$$\equiv H_{det,0} - \hat{x} \cdot \hat{F}.$$

Here, $c_{L,k}$ ($c_{R,k}$) destroys an electron state in the left (right) electrode, $\Lambda$ is the conduction-electron density of states, $\hat{m}$ denotes the number of tunnelled electrons, and the operator $Y^\dagger$ augments $\hat{m}$ by one. $\eta$ parametrizes the sensitivity of the transmission phase to $\hat{x}$, and will in general be nonzero. We consider both the cases of a pure dc voltage $V$, and a pure ac voltage $V_{ac}(t) = V \cos \nu t$. Note that the tunneling Hamiltonian itself acts as a random back-action force $\hat{F}$ on the oscillator.[9] We will describe our system by a reduced density matrix $\rho(m; x, x'; t) \equiv \langle x | \hat{\rho}(m; t) | x' \rangle$ which tracks the state of the oscillator and $m$, the number of electrons which have tunneled through the junction. In general, the evolution of $\hat{\rho}$ will be given by a Dyson-type equation:

$$\frac{d}{dt}\hat{\rho}(m,t) = -\frac{i}{\hbar}[H_0, \hat{\rho}(m;t)] + \int_{t_0}^t dt' \sum_{m'} \check{\Sigma}(m,m';t-t')$$

$$\circ [U_0(t-t')\hat{\rho}(m';t')U_0^\dagger(t-t')]. \quad (1)$$

Here, $U_0$ is the evolution operator corresponding to the unperturbed (zero-tunneling) Hamiltonian, and we have written the self-energy $\Sigma$ as a super-operator.

We will consider the simplest case of weak tunneling, and keep only self-energy terms which are lowest order in the tunneling. $\Sigma$ is only nonvanishing if $m' = m$ or $m' = m \pm 1$; these two types of contributions correspond to "scattering out" and "scattering in" terms in a kinetic equation, and are given by the diagrams shown in Fig. 1. These diagrams correspond to standard tunneling bubbles,[17] the only difference being that the tunneling vertices can contain an $\hat{x}$ operator. If $\hat{x}$ appears at the $t'$ end of a graph for $\Sigma(t,t')$, $\hat{x}$ will evolve during the duration of the tunneling event. As a result, the
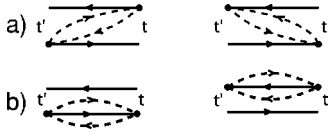
FIG. 1. Diagrams for the a) scattering in, and b) scattering out terms in the self-energy $\check{\Sigma}(t-t')$. The solid lines represent the forward and backwards Keldysh contours; the dashed lines are conduction electron propagators. The solid black vertices correspond to $\tau_0 + \tau'\hat{x}$. Note that an $\hat{x}$ operator appearing in a vertex at time $t'$ will evolve during the tunneling event.

self-energy $\check{\Sigma}$ has terms involving $\hat{p}$, and the final form of $\Sigma$ we obtain *does not* correspond to the oscillator-free case with $\hat{x}$ dependent rates. We also include perturbatively the effects of a high-temperature Ohmic heat bath ($k_B T_{bath} \gg \hbar\Omega$, with $\Omega$ being the oscillator frequency) on the oscillator using a Caldeira-Leggett description[18] and the lowest-order Born diagrams in the self-energy.

Finally, we specialize to the case where the voltage $V$ is much larger than $\hbar\Omega/e$, so that inelastic tunnel events which excite the oscillator are not suppressed. For weak tunneling and small ac frequency $\nu$, we may then make a Markov approximation in Eq. (1): $U_0(t-t')\hat{\rho}(m';t')U_0^\dagger(t-t') \to \hat{\rho}(m';t)$, which is equivalent to replacing $U_0(\tau)$ with the full evolution operator $U(\tau)$. We are assuming that over the short time scales relevant to tunneling, one can describe the dynamics of the density matrix by its zero-tunneling evolution; the omitted terms here are formally higher order in the tunneling. Fourier-transforming in the $m$ index, $\hat{\rho}(k;t) = \Sigma_{m=-\infty}^{\infty} e^{ikm}\hat{\rho}(m;t)$, Eq. (1) becomes

$$\frac{d}{dt}\hat{\rho}(k;t) = -\frac{i}{\hbar}[H_0 - \bar{F}(t,\eta)\hat{x},\hat{\rho}] - i\left(\frac{\gamma_0+\gamma}{\hbar}\right)[\hat{x},\{\hat{p},\hat{\rho}\}]$$
$$-\left(\frac{D_0+D(t)}{\hbar^2}\right)[\hat{x},[\hat{x},\hat{\rho}]] + \sum_{\sigma=+,-}\left(\frac{e^{i\sigma k}-1}{(\tau')^2}\right)$$
$$\times\left(\frac{2D_\sigma(t)}{\hbar^2}(\tau_0+e^{i\sigma\eta}\tau'\hat{x})\hat{\rho}(\tau_0+e^{-i\sigma\eta}\tau'\hat{x})\right.$$
$$+i\frac{\gamma_\sigma(t)}{\hbar}[\tau_0\tau'(e^{i\sigma\eta}\hat{p}\hat{\rho}-e^{-i\sigma\eta}\hat{\rho}\hat{p})$$
$$\left.+(\tau')^2(\hat{p}\hat{\rho}\hat{x}-\hat{x}\hat{\rho}\hat{p})]\right), \tag{2}$$

where $\gamma_0$ is the intrinsic damping coefficient associated with the equilibrium bath, $D_0 = 2M\gamma_0 k_B T_{bath}$ is the corresponding diffusion constant, and $\sigma = +(-)$ labels contributions from forward (backwards) tunneling. The detector-dependent diffusion constant $D(t) = \Sigma_\sigma D_\sigma(t)$ and damping coefficient $\gamma(t) = \Sigma_\sigma \gamma_\sigma(t)$ are given by

$$\gamma_\sigma(t) = \frac{\hbar}{2M\Omega}\left(\frac{\tau'}{\tau_0}\right)^2\left(\frac{\Gamma_\sigma(t,\hbar\Omega)-\Gamma_\sigma(t,-\hbar\Omega)}{2}\right), \tag{3}$$

$$D_\sigma(t) = \frac{\hbar^2}{4}\left(\frac{\tau'}{\tau_0}\right)^2[\Gamma_\sigma(t,\hbar\Omega)+\Gamma_\sigma(t,-\hbar\Omega)], \tag{4}$$

while $\bar{F}(t,\eta) = \sin\eta(\tau'/\tau_0)\Sigma_\sigma 2\sigma D_\sigma(t)/\hbar$ is the average back-action force exerted on the oscillator. $\Gamma_{\pm}(t,E)$ are the $\tau'=0$ finite temperature forward and backwards inelastic tunneling rates involving an absorbed energy $E$; these rates are time-independent in the case of a dc voltage. Note that we have neglected self-energy terms which renormalize the oscillator Hamiltonian; these are unimportant in the weak-tunneling limit we consider.

Equation (2) yields a compact description of the coupled detector-oscillator system; it is a generalization of an equation first derived (via an alternate approach) by Mozyrsky *et al.*[11] to an *arbitrary* detector in the tunneling regime, including the possibility of an *x*-dependent tunnelling phase, a nonlinear junction I–V, a time-dependent bias voltage, and intrinsic oscillator damping. Taking $k=0$ yields the equation for the reduced-density matrix of the oscillator, and (cf. Ref. 11) has the Caldeira-Leggett form for a forced, damped oscillator in the high-temperature regime.[18] In what follows, we focus for simplicity on the case of $T=0$ in the tunnel junction, and on $\eta=0$, which ensures $\bar{F}=0$.

*Shot Noise*. Equation (2) can be used to calculate the full counting statistics of tunneled charge as a function of time. By focusing on the time-dependence of the reduced second moment $\langle\langle m^2(t)\rangle\rangle$ (i.e., variance), it is possible to calculate the symmetrized frequency-dependent current noise using the MacDonald formula.[19] In the case of an ac bias voltage, the noise is a function of two times. We focus on the part that is independent of the average time coordinate, a quantity directly accessible in experiment. It is given by a modified version of the MacDonald formula:

$$S_I(\omega) = 2e^2\omega\int_0^\infty dt \sin\omega t\int_0^{2\pi}\frac{d\phi}{2\pi}\cdot\partial_t\langle\langle m^2(t,\phi)\rangle\rangle, \tag{5}$$

where $\phi$ is the initial phase of the ac voltage.

*dc bias*. For a dc biased normal-metal junction at $T=0$, the tunneling rates are given by $h\Gamma_\sigma(t,E) = (\tau_0)^2(\sigma eV+E)\Theta(\sigma eV+E)$. Equations (3) and (4) yield $\gamma = \hbar\tau'^2/(4\pi M)$ and $k_B T_{eff} = eV/2$.[11] We find from Eqs. (2) and (5) that the current noise may be written as $S_I(\omega) = 2e\langle I\rangle + \Delta S_I$, where the first term corresponds to purely Poissonian statistics, and the second term is a correction arising from correlations between the motion of the oscillator and the number of tunneled electrons:

$$\Delta S_I(\omega) = \frac{4e^3V}{h}\omega\int_0^\infty dt \sin\omega t((2\tau_0\tau')\langle\langle\hat{x}(t)\cdot m(t)\rangle\rangle$$
$$+(\tau')^2\langle\langle\hat{x}^2(t)\cdot m(t)\rangle\rangle). \tag{6}$$

Physically, the covariances appearing above arise from the *x*-dependence of the tunneling probability- if $m(t)$ is larger than average, then it is likely that $x(t)$ and $x^2(t)$ are also larger than average. These covariances can be calculated directly from Eq. (2), and obey simple classical equations corresponding to a forced, damped harmonic oscillator. Con-

sider first the contribution from $\langle\langle x \cdot m \rangle\rangle$ in Eq. (6), which is leading order in $\tau'$:

$$\Delta S_I(\omega)|_1 = \frac{e^3 V}{h}(2\tau_0\tau')^2\left(\frac{eV}{h} - \frac{\Omega}{4\pi}\frac{(\Delta x_0)^2}{\langle x^2 \rangle}\right)S_x(\omega), \quad (7)$$

where $\langle x^2 \rangle \simeq eV/2M\Omega^2$ is calculated using the stationary solution of Eq. (2), $S_x(\omega) = 8\gamma_{tot}\Omega^2\langle x^2 \rangle/[(\omega^2-\Omega^2)^2+4\gamma_{tot}^2\omega^2]^{-1}$ is the spectral density of oscillator $x$ fluctuations obtained from Eq. (2), and $(\Delta x_0)^2 = \hbar/(2M\Omega)$ is the zero-point uncertainty in the oscillator position. The first term in Eq. (7) is *exactly* the answer expected (to lowest order in $\tau'$) from a simple picture of a classically fluctuating junction conductance (i.e. $\Delta S_I(\omega) = V^2 S_G(\omega)$, where $S_G(\omega)$ is the spectral density of conductance fluctuations, and is in turn determined by $S_x(\omega)$). Equivalently, if we think of our junction as an $x$-to-$I$ amplifier having a gain $\lambda = 2e^2 V\tau_0\tau'/h$, this first term corresponds to simply amplifying up the fluctuations of the oscillator: $\Delta S_I = \lambda^2 S_x$. Equation (7) yields a peak in $S_I(\omega)$ at $\omega = \Omega$; keeping only the leading term in $V$, the ratio of the peak-height to the background Poissonian noise (i.e., the S/N ratio) is

$$\frac{\Delta S_I(\omega = \Omega)}{2e\langle I \rangle} = 4\tau_0^2\left(\frac{eV}{h\gamma_{tot}}\right)\frac{\alpha^2}{1+\alpha^2} \leqslant 4\left(\frac{\tau_0}{\tau'}\right)^2\frac{2MeV}{\hbar^2}, \quad (8)$$

where $\alpha^2 = \tau'^2\langle x^2 \rangle/\tau_0^2$, $\gamma_{tot} = \gamma_0 + \gamma$. Note that if $\alpha$ is small, there will be no sizeable signature of the oscillator in the average current (i.e., $\delta\langle I \rangle/\langle I \rangle_0 \simeq \alpha^2$), but there may nonetheless be a large peak in the noise if $eV/(h\gamma_{tot})$ is large. The upper bound in Eq. (8) corresponds to the optimal scenario, where there is no intrinsic (detector-independent) damping, and $\alpha \gg 1$. The maximum S/N is determined by $eV$ and the sensitivity $\tau'/\tau_0$, and can be arbitrarily large. Due to the dependence on $\gamma$, the maximum S/N is *inversely* proportional to the detector sensitivity $\tau'/\tau_0$. Note the marked difference from experiments attempting to detect coherent qubit oscillations in the detector current noise,[3] where back-action effects limit the S/N to a maximum of 4.

We turn now to the second term in Eq. (7), which is a lower-order in $V$ quantum correction to the classical result. It would appear to cause $\Delta S_I|_1$ to vanish in the limit $eV \to \hbar\Omega/2$, $\langle x^2 \rangle \to (\Delta x_0)^2$, i.e., it suppresses a zero-point contribution to $\Delta S_I|_1$. (Of course, we cannot rigorously take this limit, as Eq. (2) is strictly only valid for $eV \gg \hbar\Omega$.) A similar result was found for the average current $\langle I \rangle$ in Ref. 11, where a similar offset term could be traced to the inherent asymmetry between events in which energy is absorbed from the oscillator, versus those in which it is emitted to the oscillator. In the present case, the quantum correction to the noise in Eq. (7) can be given a classical interpretation- it arises from correlations between the intrinsic shot noise of the detector, and the back-action force $\hat{F}$ acting on the oscillator. If there are such correlations, we would expect classically:

$$\Delta S_I(\omega) = \lambda^2 S_x(\omega) + 2\lambda\,\text{Re}[g(-\omega)S_{IF}(\omega)], \quad (9)$$

where $S_{IF}(\omega)$ is the symmetrized cross-correlator between the junction current and back-action force, and $g(\omega)$ is the oscillator response function. Note that the second term above

is $\propto V$, while the first is $\propto V^2$. Inversion symmetry forces $\text{Re}\,S_{IF}$ to vanish;[5,6] however, at finite $\omega$, $\text{Im}\,S_{IF}$ is nonzero. Consequently, the second term above is nonzero; a direct perturbative calculation (assuming a thermal state for the oscillator) shows that this term corresponds to the second term in Eq. (7). Thus, *quantum corrections to the noise, which suppress zero-point contributions, can be associated with classical out-of-phase correlations between the random back-action force and the intrinsic detector output noise.*

Finally, we return to Eq. (6) and examine the contribution from $\langle\langle x^2 \cdot m \rangle\rangle$, a term which is higher-order in $\tau'$. One finds:

$$[\Delta S_I(\omega)]_2 = \frac{e^3 V}{h}(\tau')^4\left(\frac{eV}{h} - \frac{\Omega}{2\pi}\frac{(\Delta x_0)^2}{\langle x^2 \rangle}\right)$$
$$\times \int\frac{d\omega'}{2\pi}S_x(\omega')S_x(\omega-\omega'). \quad (10)$$

Again, the first term above agrees with the expectation for a classically fluctuating junction conductance; it yields peaks in $S_I$ at $\omega = 0$ and $\omega = 2\Omega$. The second term is a quantum correction, completely analogous to that found for $\Delta S_I|_1$.

*ac bias*. We now consider an ac bias voltage $V_{ac}(t) = V\cos(\nu t)$, where $eV \gg \hbar\nu, \hbar\Omega$. In the limit of small $\nu$, it is possible to derive a simple expression for the time-dependent tunneling rates.[20] Defining $h\widetilde{\Gamma}(E) = (\tau_0)^2 E \cdot \Theta(E)$, we have $\Gamma_\sigma(t,E) = \Sigma_{n=0}^\infty(1-\delta_{n,0}/2)\sigma^n\Gamma_\sigma^{(n)}(E)\cos n\nu t$, with:

$$\Gamma_\sigma^{(n)} = \sum_\pm\int_0^\pi\frac{d\theta}{\pi}\cos(n\theta)\widetilde{\Gamma}\left(eV\cos\theta + E \pm \frac{n\hbar\nu}{2}\right). \quad (11)$$

Using Eqs. (3) and (4), we find that the damping coefficient $\gamma$ of the oscillator is time-independent and identical to that in the dc case, whereas the diffusion constant is time-dependent and contains higher harmonics of the ac frequency $\nu$. Writing $D(t) = 2M\gamma k_B T_{eff}(t)$, we have to a good approximation:

$$k_B T_{eff}(t) = \frac{eV}{\pi}\left[\sum_{n=0}^{eV/\hbar\nu}\left(\frac{2(-1)^n}{1-(2n)^2}\right)\cos(2n\nu t) - 1\right]. \quad (12)$$

The small but finite photon frequency $\nu$ prevents higher harmonics from contributing to $T_{eff}$; without it, we would have simply $k_B T_{eff}(t) = V|\cos\nu t|/2$, which tends to zero twice each period. With the finite cut-off included, the minima of $k_B T_{eff}(t)$ are $\simeq \hbar\nu$. The time-dependence of $T_{eff}(t)$ implies that the position variance $\langle x^2(t) \rangle$ of the oscillator will be time-dependent; as we show, this has a direct influence on the noise and the average current. For the latter quantity, we find:

$$\langle I(t) \rangle = \frac{e^2 V}{h}\cos(\nu t)[\tau_0^2 + (\tau')^2\langle x^2(t) \rangle] - \Delta I(t),$$

where the quantum correction is approximately $\Delta I(t) \simeq e\gamma\cdot\text{sgn}[\cos\nu t]$. Turning to the noise, we may again decompose $S_I(\omega)$ into a frequency-independent part and a term arising from correlations between $x(t)$ and $m(t)$: $S_I(\omega) \equiv (\Omega/2\pi)\int_0^{2\pi/\Omega}d\bar{t}S_I(\bar{t},\omega) = S_I^a + \Delta S_I(\omega)$. For the frequency-independent contribution $S_I^a$, we find:
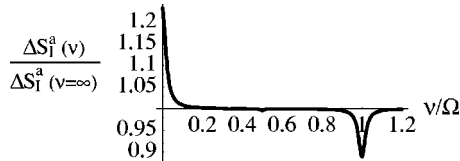
FIG. 2. Oscillator contribution to $S_I^a$, the frequency-independent part of the shot noise (i.e., second term in Eq. (13)), versus the ac voltage frequency $\nu$, for $\Omega/\gamma=50$, $\gamma_0=0$, and $eV \gg \hbar\nu, \hbar\Omega$. The maximum suppression of this term at $\nu=\Omega$ (over its $\nu \to \infty$ limit) is by 8/9.

$$S_I^a = \frac{4e^2}{h}\left[\frac{eV}{\pi}\tau_0^2 + (\tau')^2\left(\overline{k_B T_{eff}(t)\langle x^2(t)\rangle} - \frac{\Omega}{2\pi}(\Delta x_0)^2\right)\right],$$

(13)

where the bar indicates a time-average. The first term is the standard result for the shot noise of an ac-biased junction.[21] The second term indicates that the time-dependence of $\langle x^2(t)\rangle$ [calculated from Eq. (2)] makes a frequency-independent contribution to the noise. For $\nu \gg \Omega$, $\langle x^2 \rangle$ responds only weakly to the time-dependence of $T_{eff}(t)$, whereas for $\nu \sim \Omega$, the response becomes appreciable and 180 degrees out-of-phase with $V(t)$. If in addition $\gamma_0 \ll \gamma$, one finds a resulting *suppression* of the oscillator's contribution to $S_I^a$; this is shown in Fig. 2. Small resonances also occur when $\Omega$ is a multiple of $\nu$. The oscillator modification of $S_I^a$ is not captured by the classical picture of a fluctuating conductance.

Finally, the frequency-dependent contribution $\Delta S_I(\omega)$ to the noise, which arises from correlations between $x(t)$ and $m(t)$, takes the simple form:

$$\Delta S_I(\omega) = \frac{1}{4}\sum_{\pm} \Delta S_I(\nu \pm \omega)|_{dc}\left[1 + O\left(\frac{\hbar\Omega}{eV}\right)\right],$$

(14)

where the omitted terms correspond to "quantum corrections" of the sort previously discussed. Without these, Eq.
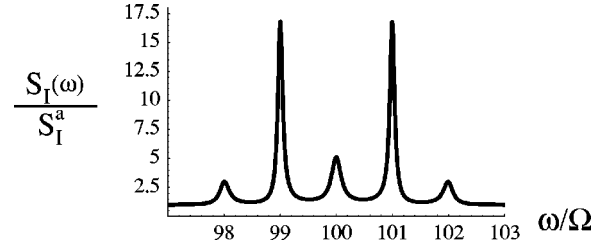


FIG. 3. Full shot noise $S_I(\omega)$ for an ac bias voltage of frequency $\nu=100\,\Omega$, including the effects of correlations between $x(t)$ and $m(t)$ (cf. Eq. (14)). We have chosen $eV=100\hbar\Omega$, $\alpha^2=1$, $(\gamma_0+\gamma)=\Omega/20$, and $\tau_0=0.1$. The $y$ axis is scaled by the value of the $\omega$-independent part of the noise.

(14) is precisely the answer expected for a fluctuating classical conductance—one needs to simply shift the noise in the dc case up to the frequency $\nu$. In contrast, the quantum corrections to $\Delta S_I$ for ac bias are not simply given by shifting the corresponding terms found for dc bias—one finds that the quantum corrections are larger in the ac case by a factor of $4/\pi$. The effect of $\Delta S_I(\omega)$ on the full noise is shown in Fig. 3.

In conclusion, we have presented a fully quantum mechanical calculation of the frequency-dependent current noise of a tunnel junction displacement detector, for both the cases of dc and ac voltage bias. The oscillator can lead to large effects in the shot noise, even if the coupling to the detector is weak; moreover, these effects cannot be completely described using a classical picture of a fluctuating junction conductance.

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