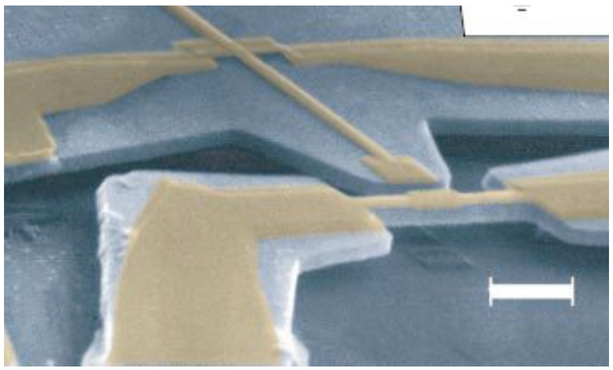
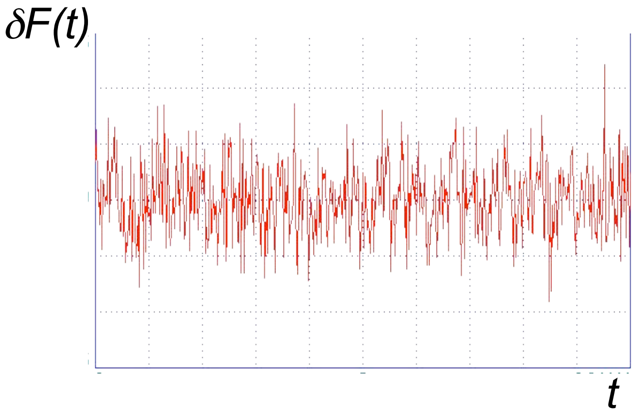
Quantum Noise and Quantum Measurement

(APS Tutorial on Quantum Measurement)



Aashish Clerk

McGill University



(With thanks to S. Girvin, F. Marquardt, M. Devoret)

- Use quantum noise to understand quantum measurement…

NSERC CRSNG

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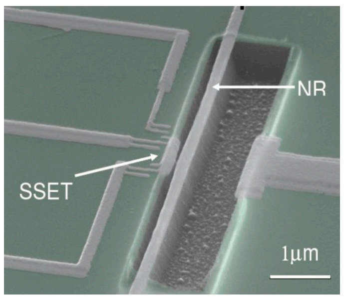
Quantum Measurement & Mesoscopic Physics

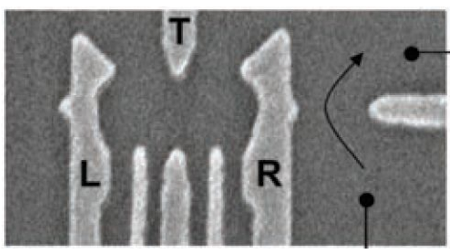
- ***Quantum measurement*** relevant to many recent expts…



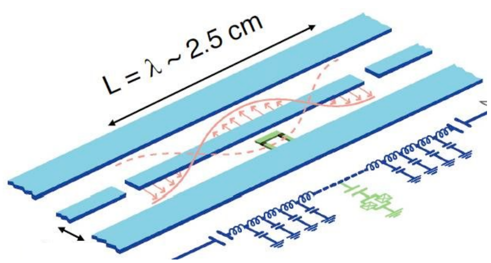
(K. Schwab)

(K. Lehnert)

Quantum electro-mechanical systems…

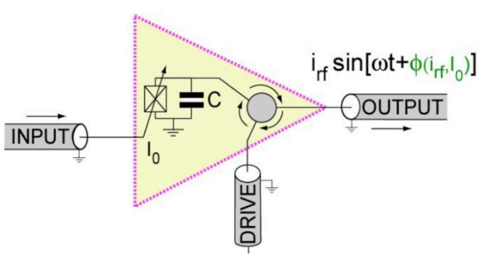


(C. Marcus, J. Petta)

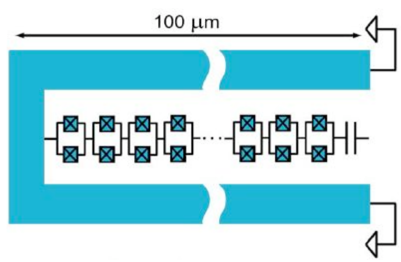


(R. Schoelkopf)

Qubit + readout experiments…



(M. Devoret, I. Siddiqi)

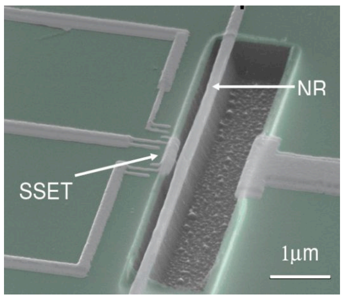


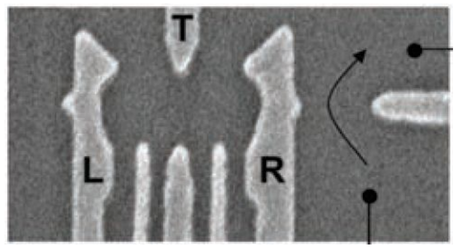
(K. Lehnert)

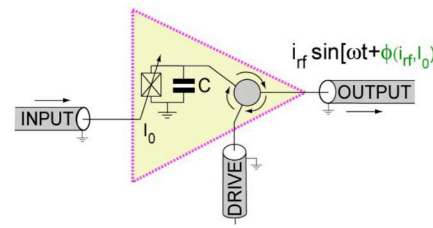
Quantum-limited & back-action evading amplifiers…

Quantum Measurement & Mesoscopic Physics

- ***Quantum measurement*** relevant to many recent expts…



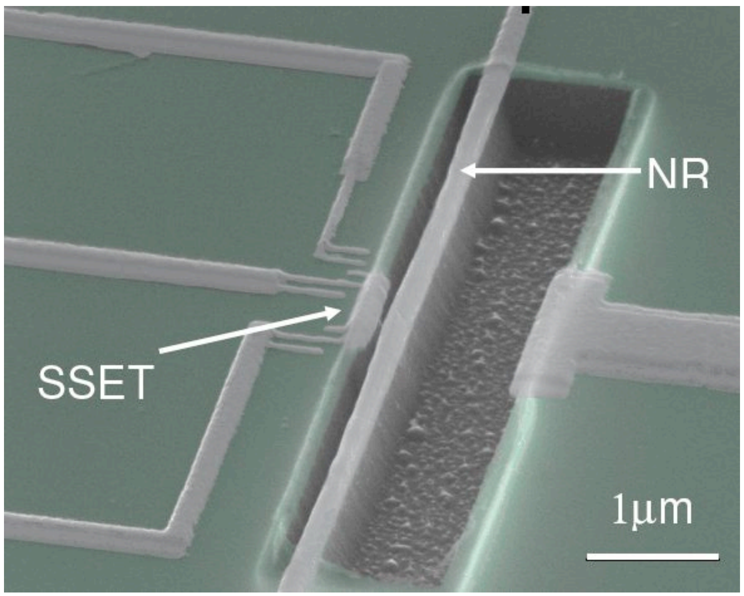




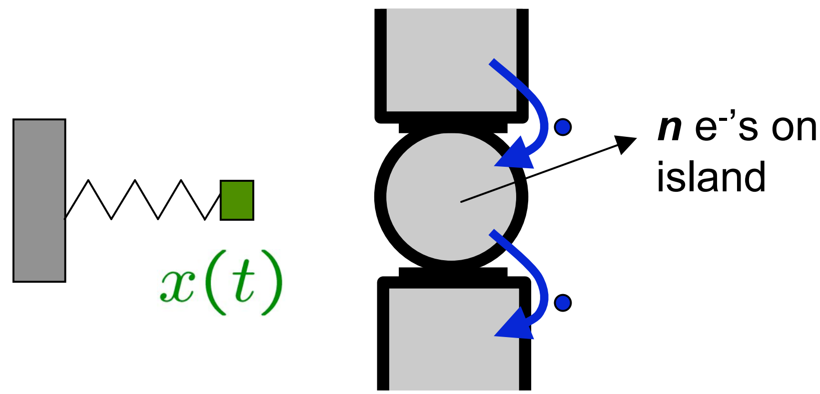
- **Issues?**
  1. *How do we describe the "back-action" of a detector?*
     - Detector is quantum and out-of-equilibrium
  2. *What is the "quantum limit"?*
     - How do we reach this ideal limit?
  3. *Conditional evolution?*
     - What is the state of the measured system given a particular measurement record?

Weak Continuous Measurements

- Information only acquired gradually in time…
- Need to average to reduce the effects of noise
- *e.g. oscillator measured by a single-electron transistor:*



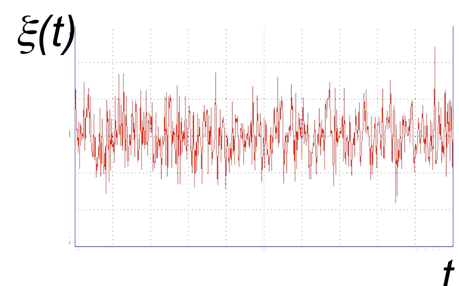
(K. Schwab group, Cornell)



$$H_{int} = A\hat{x} \cdot \hat{n}$$

$$I(t) = \lambda x(t) + \xi(t)$$

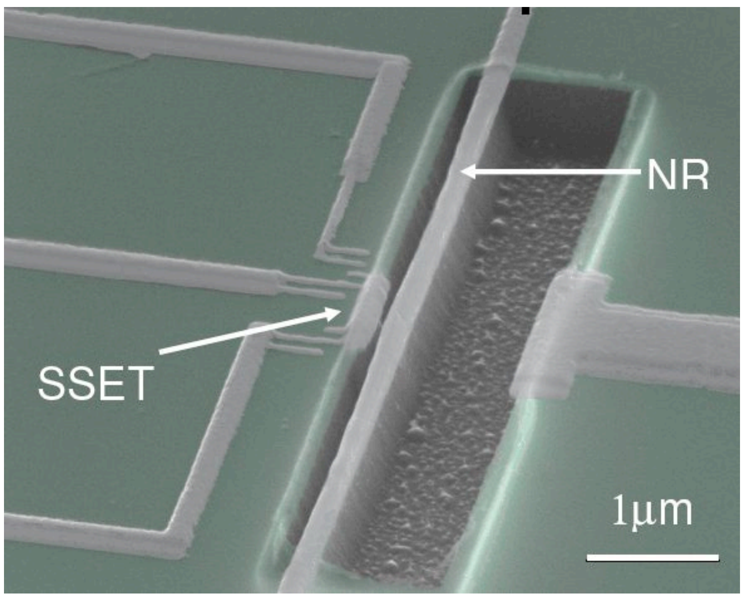
$$\lambda = \frac{dI}{dU} \times \frac{dU}{dx}$$



Weak Continuous Measurements

- ***Not*** measuring instantaneous x(t);
rather "quadrature amplitudes":

$$x(t) = X(t) cos \omega_M t + Y(t) \sin \omega_M t$$



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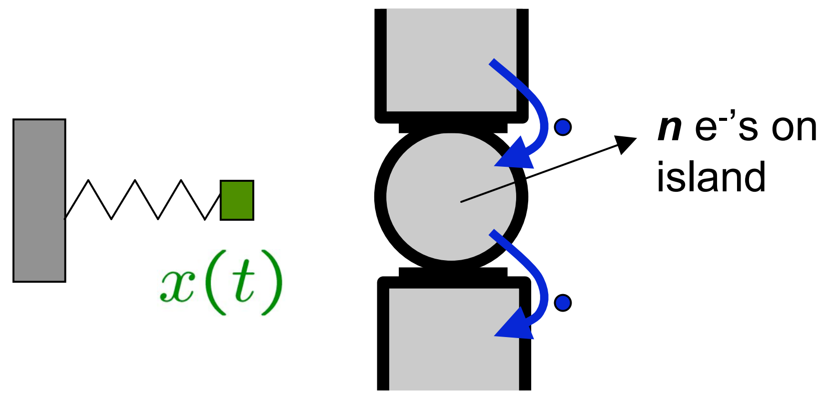

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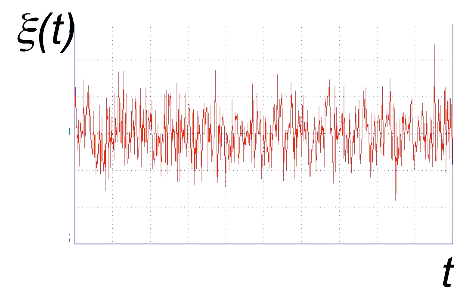
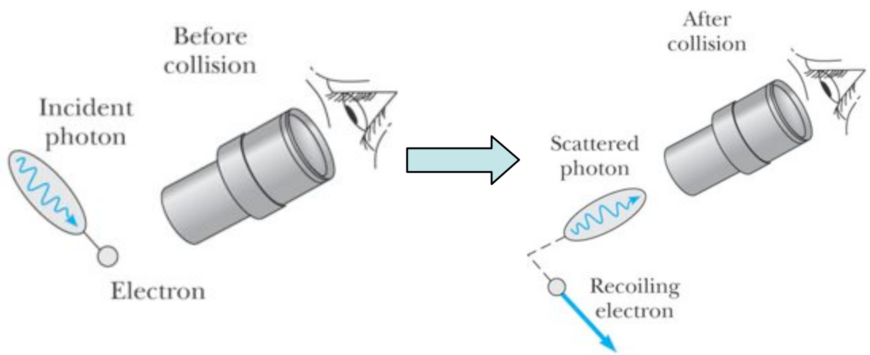


Quantum Limits?

- ***Naïve:*** for a more precise measurement, just increase coupling, hence λ.... $I(t) = \lambda x(t) + \xi(t)$
- ***BUT:*** *back-action* puts a limit to how much you can do this!

Heisenberg microscope



$$\Delta x \cdot \Delta p \geq \frac{h}{2}$$

- Measure x ⇒ disturb p
- This messes up meas. of x at later times

$$\Delta x(\delta t) = \Delta x(0) + \delta t \frac{\Delta p}{m}$$

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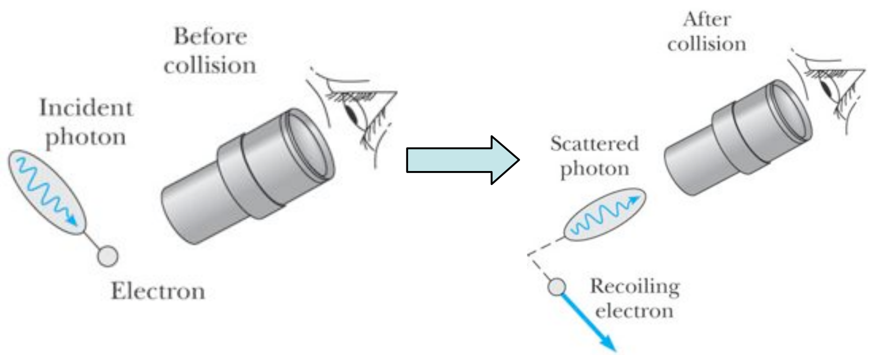

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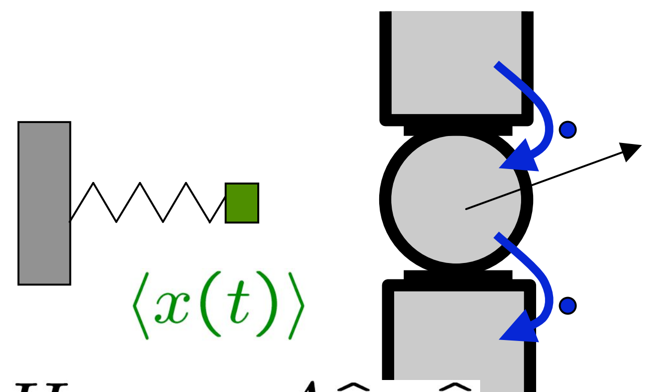
SET position detector?



$$H_{int} = A\hat{x} \cdot \hat{n}$$

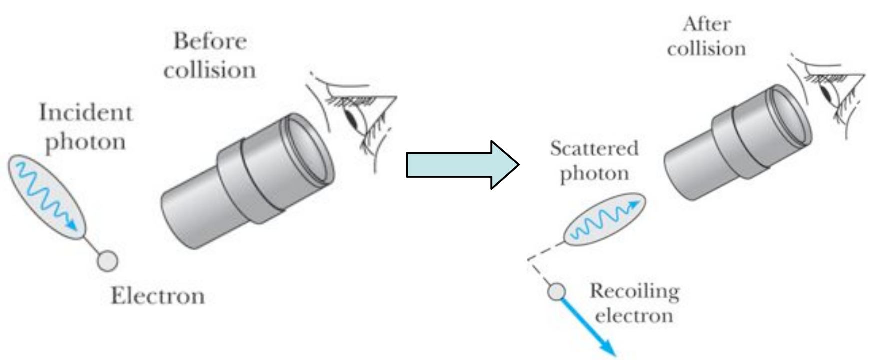
- *A·n* is a fluctuating force on oscillator… will cause x to fluc.

$$\delta I(t) = \lambda \delta x(t) + \xi(t)$$

Quantum Limits?

- ***Naïve:*** for a more precise measurement, just increase coupling, hence λ.... $\hat{I}(t) = \lambda\langle x(t)\rangle + \xi(t)$
- ***BUT:*** *back-action* puts a limit to how much you can do this!

Heisenberg microscope



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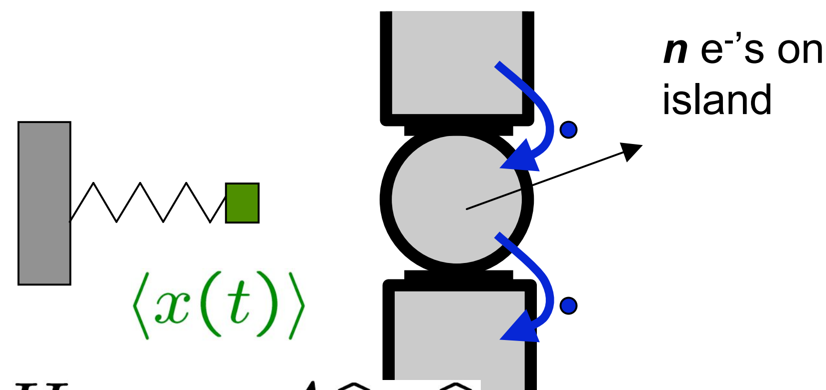


$$H_{int} = A\hat{x} \cdot \hat{n}$$

- *A·n* is a fluctuating force on oscillator… will cause x to fluc.

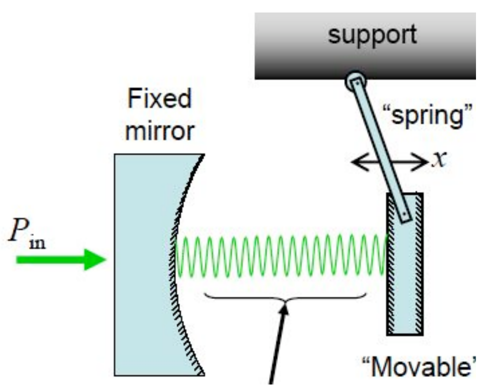
How do we describe back-action? Is it "ideal" or not?

Quantum Noise Approach

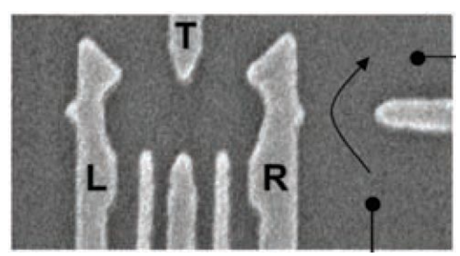
- What does back-action do to the oscillator?
- Is it as small as allowed by quantum mechanics?
  - *Need to understand the (quantum) noise properties of the detector's back-action force..*

$$H = H_{system} + H_{detector} - \hat{x} \cdot \hat{F}$$

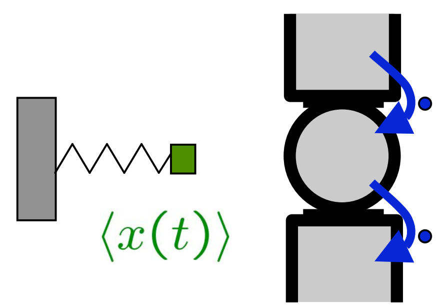
- Force exerted by detector described by an operator ***F***



F ~ cavity photon number



F ~ charge in QPC



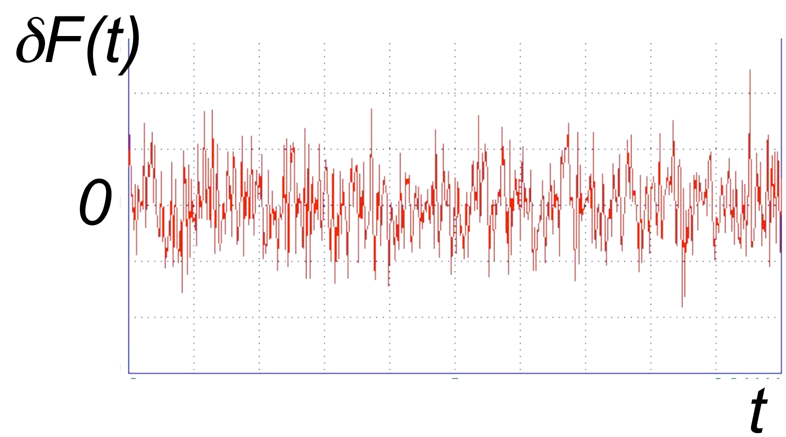
F ~ n

Basics of Classical Noise

- Start by thinking of the noisy force *F(t)* classically…

$$F(t) = \bar{F} + \delta F(t)$$

$$\delta F(\omega) = \frac{1}{\sqrt{T}} \int_0^T dt\, [\delta F(t)]\, e^{i\omega t}$$

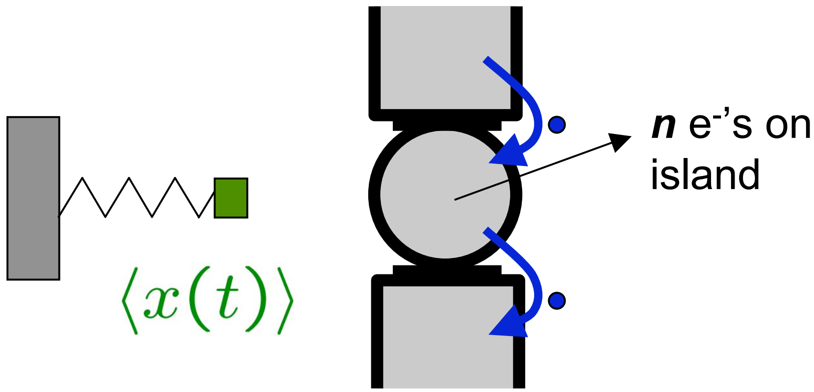


- Power spectral density: how big is the noise at a given frequency?

$$\begin{aligned} S_F(\omega) &\equiv \langle |\delta F(\omega)|^2 \rangle \\ &= \int_{-\infty}^{\infty} \langle \delta F(t) \cdot \delta F(0) \rangle\, e^{i\omega t} \end{aligned}$$

- Stationary noise? Autocorrelation only depends on time difference
- Gaussian noise? Full probability distribution set by $S_F(\omega)$

What about a noisy quantum force?



$$H_{int} = A\hat{x} \cdot \hat{n}$$
$$\hat{F} = -A\hat{n}$$

- Back-action force is a quantum operator; also described by a spectral density…

$$S_F(\omega) = \int dt e^{i\omega t} \langle \hat{F}(t)\hat{F}(0) \rangle$$

Heisenberg-picture operators

Expectation value is with respect to the density matrix describing the detector's state…

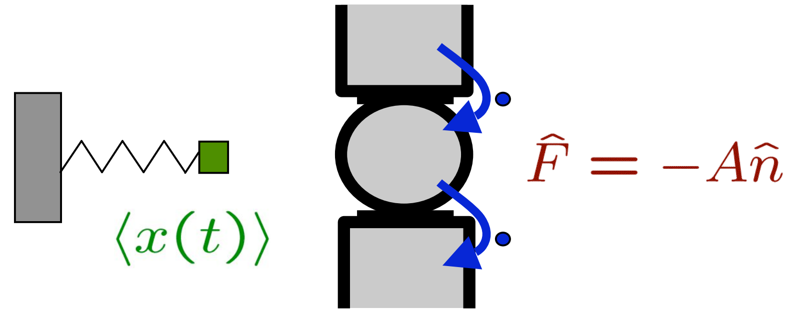
What is so quantum about quantum noise?

$$S_F(\omega) = \int dt e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle$$

1. *Zero-point fluctuations*
   - Noise does not vanish at zero temperature
   - At high frequencies, $\hbar\omega > k_B T$; noise will be much bigger than the classical prediction
2. *Positive and negative frequencies not the same!*
   - Classical: $\delta F(-\omega) = \delta F(\omega)^*$, thus $S_F(\omega) = S_F(-\omega)$
   - Quantum: ***F***(t) and ***F***(0) do not commute!
3. *Heisenberg-like quantum constraints on noise!*
   - The uncertainty principle places a rigorous lower bound on S_F

Effects of the back-action force?

- *Classical case*: use a Langevin equation:



$$m\ddot{x} = -kx - \int dt' m\gamma(t-t')\dot{x}(t') + \delta F(t)$$

damping kernel

random force

$$S_{\delta F}(\omega) = 2m\gamma(\omega)k_B T$$

- *Quantum case*: Langevin equation still holds ***if*** the detector is in equilibrium and has Gaussian noise…

$$S_{\delta F}(\omega) = m\gamma(\omega)\hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

- ***But:*** **any reasonable detector is NOT in equilibrium! What is the "effective temperature" of the detector?**

Positive versus Negative Frequency Noise?

$$S_F(\omega) \;=\; \int dt e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle \qquad S_F(\omega) \neq S_F(-\omega)$$

- Instructive to write $S_F(\omega)$ in terms of the exact eigenstates of our "bath":

$$S_F(\omega) = 2\pi \sum_{f,i} \rho_{ii} |\langle f|F|i\rangle|^2 \delta(E_f - E_i + \omega)$$

Bath density matrix

Bath energy eigenstates

- Just the Golden Rule expression for a transition rate!

$\omega>0$: absorption of $\hbar\omega$ by bath

$\omega<0$: emission of $\hbar\omega$ by bath

Effective Temperature

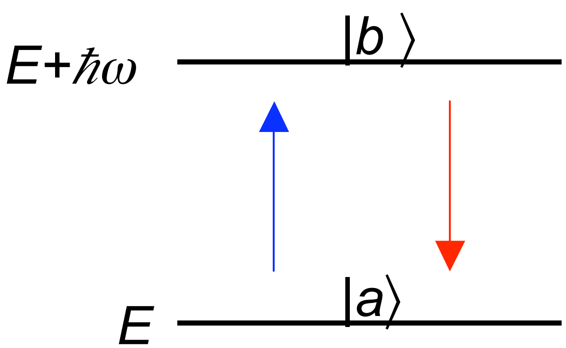
$$S_F(\omega) \;=\; \int dt e^{i\omega t}\langle \hat{F}(t)\hat{F}(0)\rangle$$

ω>0: absorption of ħω by bath

ω<0: emission of ħω by bath

$$S_F(\omega) = 2\pi \sum_{f,i} \rho_{ii} |\langle f|F|i\rangle|^2 \delta(E_f - E_i + \omega)$$

- In equilibrium, quantum noise directly tied to temperature…
- Consider the rate at which the detector makes transitions between states with energy *E* and *E*+ħω…



$$\frac{\rho_{bb}}{\rho_{aa}} \;=\; \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

Thus:

$$\frac{\text{rate to emit } \hbar\omega}{\text{rate to absorb } \hbar\omega} = \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

Effective Temperature

$$S_F(\omega) \;=\; \int dt e^{i\omega t}\langle \hat{F}(t)\hat{F}(0)\rangle$$

ω>0: absorption of ħω by bath

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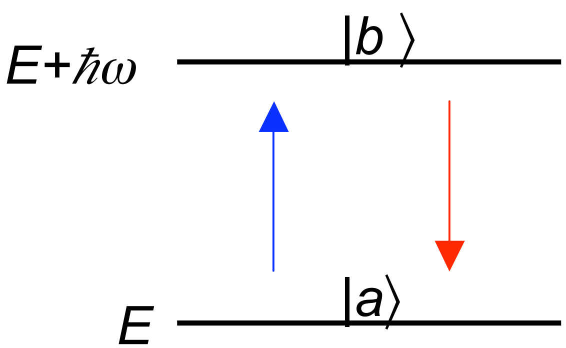


$$\frac{\rho_{bb}}{\rho_{aa}} \;=\; \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

Thus:

$$\frac{S_F(-\omega)}{S_F(\omega)} \;\equiv\; \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

Effective Temperature

$$S_F(\omega) = \int dt e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle$$

ω>0: absorption of ℏω by bath

ω<0: emission of ℏω by bath

$$S_F(\omega) = 2\pi \sum_{f,i} \rho_{ii} |\langle f|F|i\rangle|^2 \delta(E_f - E_i + \omega)$$

- In equilibrium, ratio between positive and negative noise set by temperature:

$$\frac{S_F(-\omega)}{S_F(\omega)} \equiv \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

- BUT: What if bath is *not* in equilibrium?
- Can use this ratio to *define* an effective temperature….

$$\frac{S_F(-\omega)}{S_F(\omega)} \equiv \exp\left(-\frac{\hbar\omega}{k_B T_{eff}(\omega)}\right)$$

Effective Temperature

$$S_F(\omega) = \int dt e^{i\omega t} \langle \hat{F}(t)\hat{F}(0)\rangle$$

$\omega>0$: absorption of $\hbar\omega$ by bath

$\omega<0$: emission of $\hbar\omega$ by bath

$$\frac{S_F(-\omega)}{S_F(\omega)} \equiv \exp\left(-\frac{\hbar\omega}{k_B T_{eff}(\omega)}\right)$$

- *T_{eff} in a non-equilibrium system?*
  - a measure of the asymmetry between emission and absorption
- T_{eff} is frequency-dependent?
  - the price we pay for being out-of-equilibrium!

Still… how does this relate to more usual notions of temperature?

Effective bath descriptions

- For weak coupling, can *rigorously* derive a Langevin equation

A.C., Phys. Rev. B 70 (2004); (also J. Schwinger, J. Math Phys. 2 (1960);Mozyrsky, Martin & Hastings, PRL 92 (2004))

$$m\ddot{x} = -\tilde{k}x - \int dt' m\gamma(t-t')\dot{x}(t') + \delta F(t)$$

damping kernel

random force

$$\gamma(\omega) = \frac{S_F(\omega) - S_F(-\omega)}{2m\hbar\omega} \qquad S_{\delta F}(\omega) = \frac{S_F(\omega) + S_F(-\omega)}{2}$$

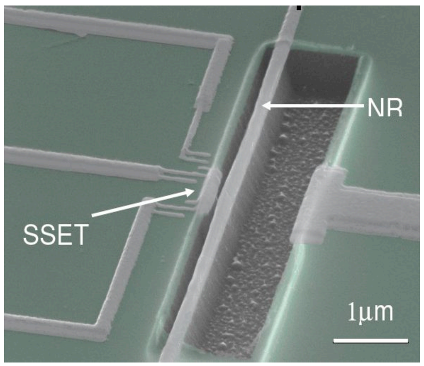
$$\bar{S}_{\delta F}(\omega) = m\gamma(\omega)\hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T_{eff}(\omega)}\right)$$

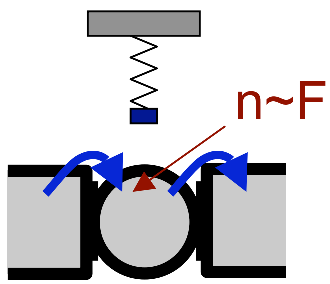
- *Generic approach:*
  - To understand how the detector acts as a “bath”, need to know its $S_F(\omega)$…
- *T_{eff}?*
  - Energy scale characterizing the difference between energy absorption and emission

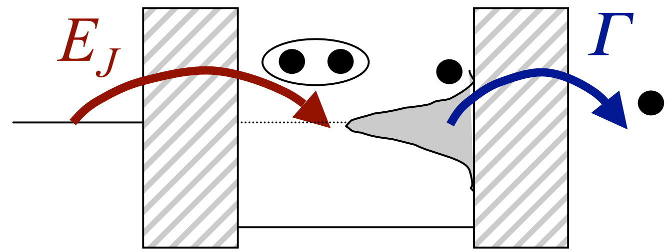
Applications of this Approach?

KEY: If we understand the quantum noise properties of our detector, we understand how it acts as a bath…

- Back-action cooling with Cooper Pairs:







$\delta = E_{final} - E_{initial}$

T_{eff} NOT set by bias voltage
Rather, by lifetime of a resonance!
(expt: V_{DS} ~ 5K, T_{eff} ~ 200 mK)

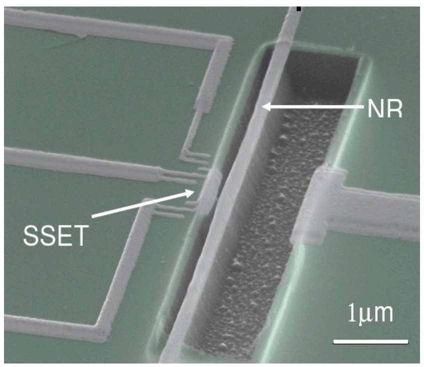
$$k_B T_{eff} = \frac{(\hbar\Gamma_a)^2 + 4\delta^2}{16\delta}$$

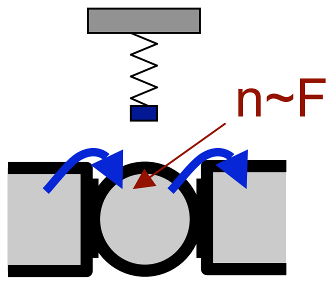
Theory: AC, Girvin & Stone, 02; AC & Bennett, 05; Blencowe, Armour & Imbers, 05
Expt: Naik et al, 2006

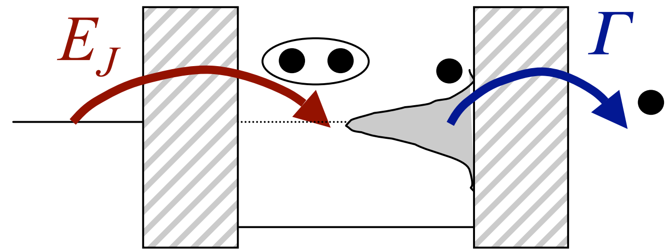
Applications of this Approach?

KEY: If we understand the quantum noise properties of our conductor, we understand how it acts as a bath…

- Back-action cooling with Cooper Pairs:







$\delta = E_{final} - E_{initial}$

- What if oscillator frequency is not small?

$$n_{osc} = \frac{1}{e^{\hbar\omega/(k_B T_{eff}(\omega))} - 1} = \frac{(\hbar\omega - \delta)^2 + (\Gamma/2)^2}{4\hbar\omega\delta}$$

- ***If $\delta = \hbar\omega$, $\hbar\omega >> \Gamma$?***

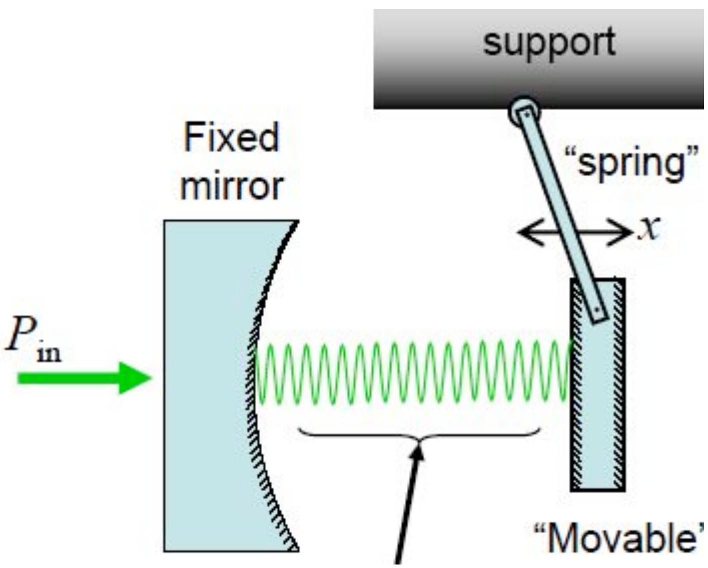
$$n_{osc} \rightarrow \left(\frac{\Gamma}{4\hbar\omega}\right)^2 \rightarrow 0$$

(AC, unpublished)

Applications of this Approach?

KEY: If we understand the quantum noise properties of our conductor, we understand how it acts as a bath…

- Back-action cooling with photons:



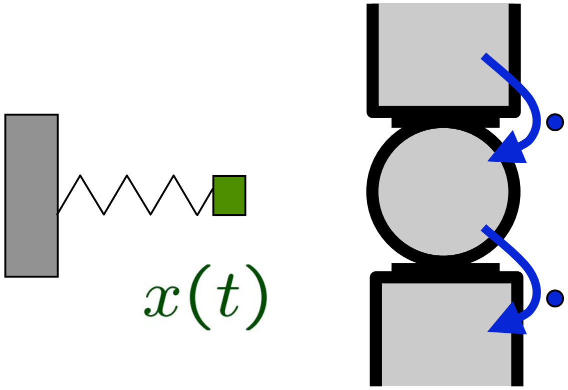
$$H_{int} = A a^\dagger a \cdot x$$

$$\bar{n}_M^O = -\frac{(\omega_M + \Delta)^2 + (\kappa/2)^2}{4\omega_M \Delta}$$

- Same expression as for Cooper pair cooling!
- Can reach ground state for large ω_m / κ…

Quantum theory: Marquardt, Chen, AC, Girvin, 07; Wilson-Rae, Nooshi, Zwerger & Kippenberg 07
Expts: Hohberger-Metzger et al., 04; Arcizet et al., 06; Gigan et al. 06; Schliesser et al. 06; Corbitt et al 07; Thompson et al. 08

Towards the Quantum Limit



$$H_{osc} = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$H_{int} = A\hat{x} \cdot \hat{n}$$

$$\begin{aligned} I(t) &= \lambda x(t) + \xi(t) \\ &= \lambda\left[x_0(t) + \tilde{\xi}(t)\right] \end{aligned}$$

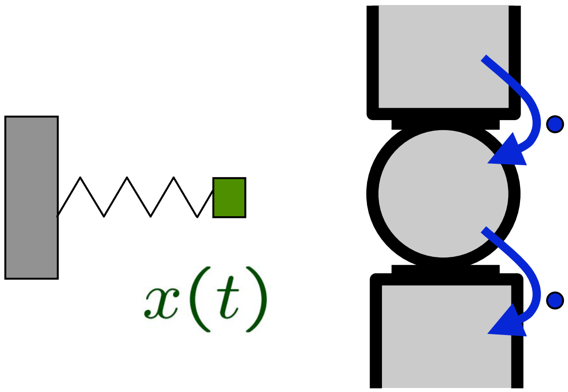
- How small can we make the added noise?

Two parts to the noise: $\tilde{\xi}(t) = \frac{\xi_0(t)}{\lambda} + \xi_{BA}(t)$

"Intrinsic" output noise:

- Present even without coupling to oscillator (e.g. shot noise)
- Make it smaller by *increasing* coupling strength…

Towards the Quantum Limit



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$$\begin{aligned} I(t) &= \lambda x(t) + \xi(t) \\ &= \lambda\left[x_0(t) + \tilde{\xi}(t)\right] \end{aligned}$$

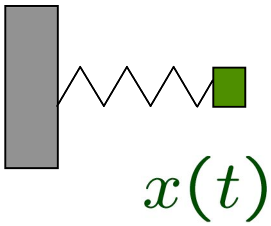
- How small can we make the added noise?

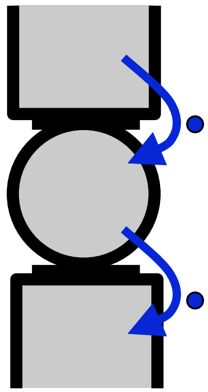
Two parts to the noise: $$\tilde{\xi}(t) = \frac{\xi_0(t)}{\lambda} + \xi_{BA}(t)$$

Back-action noise:

- Measuring x ***must*** disturb p in a random way; this leads to uncertainty in x at later times.
- Make it smaller by *decreasing* coupling strength…

Amplifier Quantum Limit





$$H_{osc} = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$H_{int} = A\hat{x} \cdot \hat{n}$$

$$\begin{aligned} I(t) &= \lambda x(t) + \xi(t) \\ &= \lambda \left[x_0(t) + \tilde{\xi}(t) \right] \end{aligned}$$

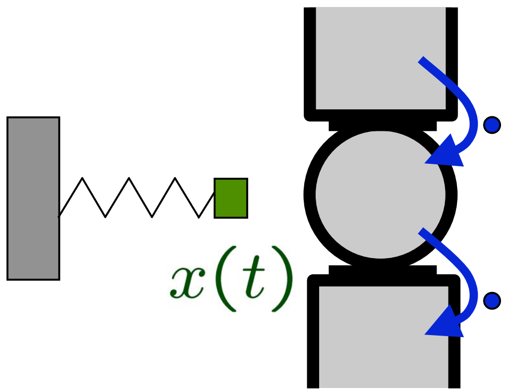
- How small can we make the added noise?

Two parts to the noise: $$\tilde{\xi}(t) = \frac{\xi_0(t)}{\lambda} + \xi_{BA}(t)$$

Quantum Limit

- If our detector has a "large" gain, then $\tilde{\xi}(t)$ *cannot be arbitrarily small*
- The *smallest* it can be is the size of the oscillator zero-point motion…

A Precise Statement of the QL



$$\langle \hat{I}(t) \rangle = \lambda \langle x(t) \rangle$$

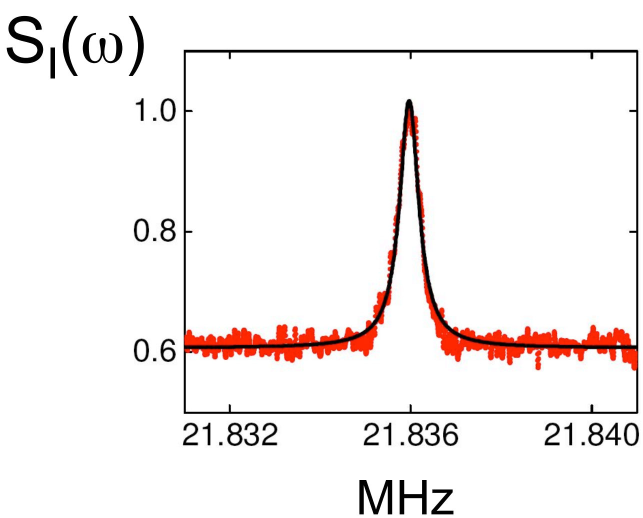
If there were no noise:

$$S_I(\omega) = \lambda^2 S_x(\omega)$$

Including noise added by detector:

$$S_I(\omega) = \lambda^2 \left[S_x(\omega) + \delta S_x(\omega) \right] + S_{\xi_0}(\omega)$$

(Ignore correlation for the moment!)



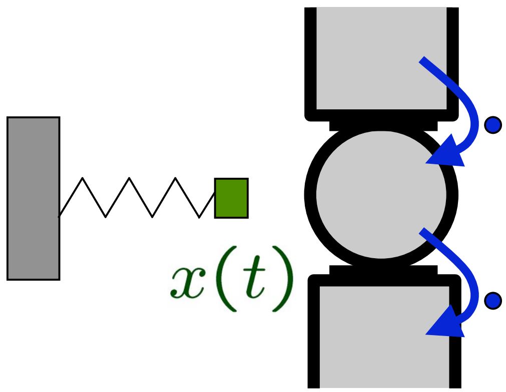
Added noise $S_{x,\mathrm{add}}(\omega)$:

$$S_{x,\mathrm{add}}(\omega) = \frac{1}{\lambda^2} S_{\xi_0}(\omega) + \delta S_x(\omega)$$

Quantum limit?

$$\begin{aligned} S_{x,\mathrm{add}}(\omega) &\geq S_{x,\mathrm{zpt}}(\omega) \\ &= (\Delta x_{\mathrm{zpt}})^2 \frac{2\Omega\gamma|\omega|}{(\omega^2 - \Omega^2)^2 + \omega^2\gamma^2} \end{aligned}$$

A loophole?



$$\left\langle \hat{I}(t) \right\rangle = \lambda \langle x(t) \rangle$$

If there were no noise:

$$S_I(\omega) = \lambda^2 S_x(\omega)$$

Including noise added by detector:

$$S_I(\omega) = \lambda^2 \left[S_x(\omega) + \delta S_x(\omega) \right] + S_{\xi_0}(\omega)$$

(Ignore correlation for the moment!)

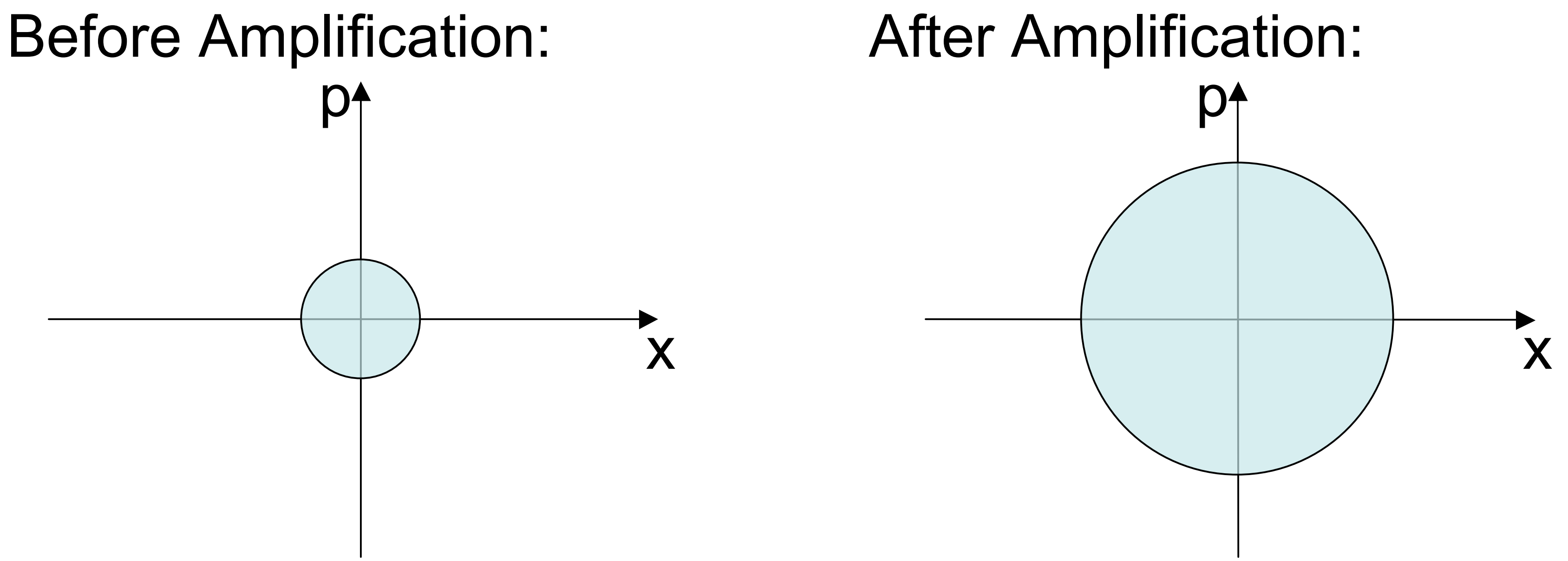
Added noise $S_{x,\text{add}}(\omega)$: $S_{x,\text{add}}(\omega) = \frac{1}{\lambda^2} S_{\xi_0}(\omega) + \delta S_x(\omega)$

WAIT: what if back-action force and "shot noise" anti-correlated?

$$\tilde{\xi}(t) = \frac{\xi_0(t)}{\lambda} + \xi_{BA}(t)$$

Could in principle have back-action, yet still have no added noise!

Why must there be added noise?

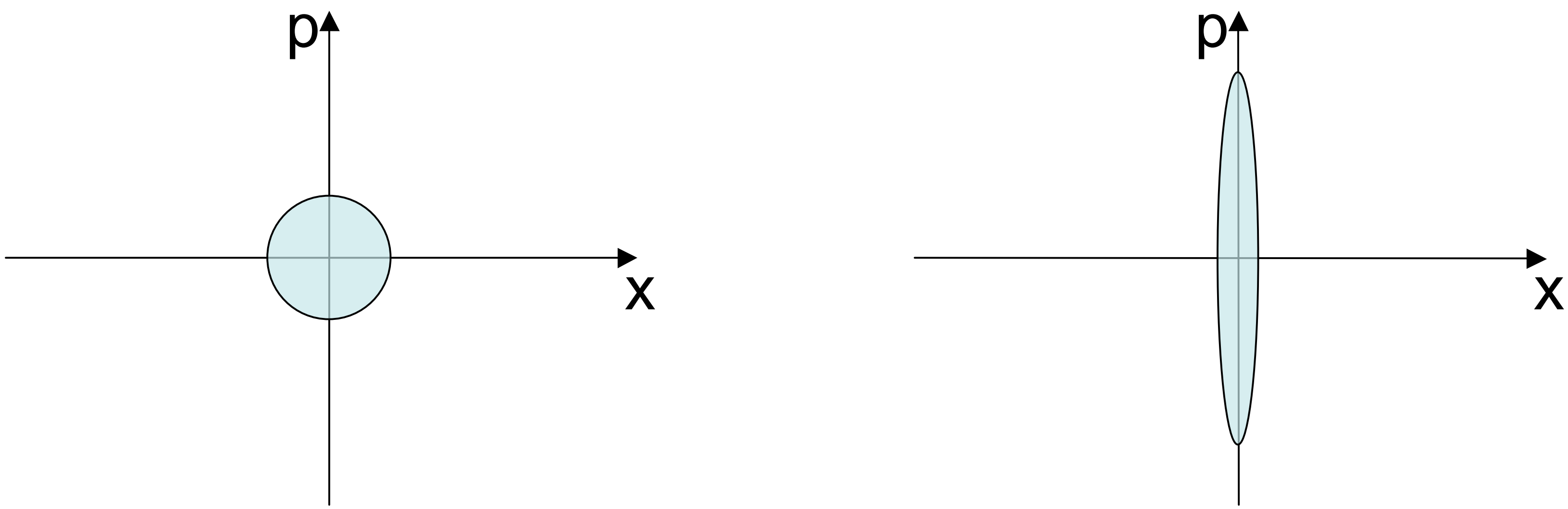


- Liouville Theorem: phase-space volume can't expand!
- Way out: there *must* be extra degrees of freedom
- Quantum: these extra degrees of freedom *must* have some noise (at the very least, zero-point noise)

(can use this to derive amplifier quantum limt: Haus & Mullen, 62; Caves 82)

Aside: Noise-Free Amplification?

Before Amplification: After Amplification:



- Can amplify one quadrature without any added noise:

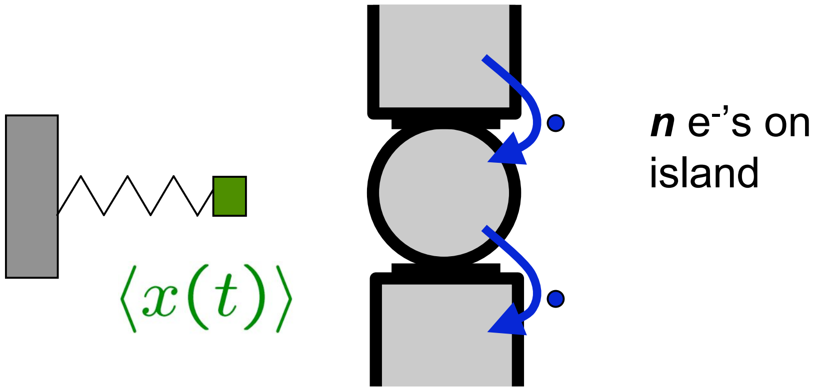
$$x(t) = X(t)\cos\Omega t + Y(t)\sin\Omega t$$

$$\Downarrow$$

$$x(t) = e^{-A}X(t)\cos\Omega t + e^{A}Y(t)\sin\Omega t$$

- Can realize this in many ways
  e.g. driven cavity coupled to osc. (AC, Marquardt, Jacobs, 08)

Detector Noise



$$\hat{I}(t) = \lambda\langle x(t)\rangle + \xi(t)$$

Noise characterized by symmetrized spectral densities:

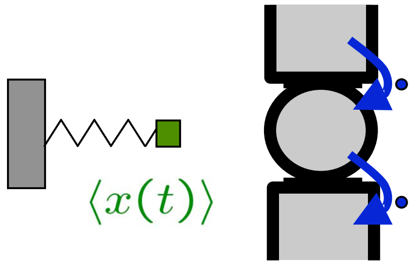
$$\bar{S}_I(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dt \left\langle \left\{ \delta\hat{I}(t), \delta\hat{I}(0) \right\} \right\rangle e^{i\omega t}$$

$$\bar{S}_F(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dt \left\langle \left\{ \delta\hat{F}(t), \delta\hat{F}(0) \right\} \right\rangle e^{i\omega t}$$

$$\bar{S}_{IF}(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dt \left\langle \left\{ \delta\hat{I}(t), \delta\hat{F}(0) \right\} \right\rangle e^{i\omega t}$$

Quantum Constraint on Noise

AC, Girvin & Stone, PRB **67** (2003)
Averin, cond-mat/031524



$$\hat{I}(t) = \lambda \langle x(t) \rangle + \xi(t)$$

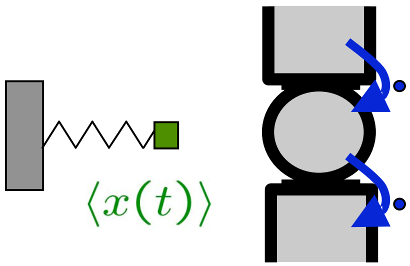
Important aspect of quantum noise:

There are quantum constraints on noise that have no classical analogue.

$$\bar{S}_I(\omega)\bar{S}_F(\omega) - [\mathrm{Re}\ \bar{S}_{IF}(\omega)]^2 \geq \left(\frac{\hbar\lambda(\omega)}{2}\right)^2$$

- ***If we have gain, we MUST in general have noise.***
- To simplify inequality, have assumed:
  - No reverse gain (if you couple to I, F is not affected)
  - λ is real

Origin of Quantum Noise Constraint



$$\hat{I}(t) = \lambda \langle x(t) \rangle + \xi(t)$$

$$\bar{S}_I(\omega)\bar{S}_F(\omega) - \left[\text{Re } \bar{S}_{IF}(\omega)\right]^2 \geq \left(\frac{\hbar\lambda(\omega)}{2}\right)^2$$

- Usual Heisenberg Uncertainty relation:

$$(\Delta A)^2(\Delta B)^2 \geq \frac{1}{4}\langle\{A, B\}\rangle^2 + \frac{1}{4}\left|\langle[A, B]\rangle\right|^2$$

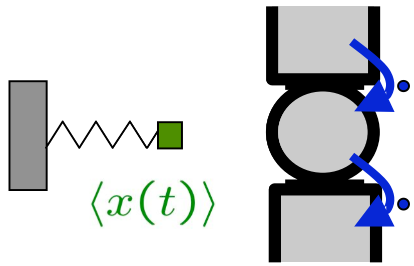
- To have gain, I(t) and F(0) can't commute for all times t!

$$\lambda(t) \equiv -\frac{i}{\hbar}\theta(t)\,\langle[I(t), F(0)]\rangle$$

- This non-commutation at different times leads directly to our quantum constraint on the noise

Quantum Constraint on Noise

AC, Girvin & Stone, PRB **67** (2003)
Averin, cond-mat/031524



$$\hat{I}(t) = \lambda \langle x(t) \rangle + \xi(t)$$

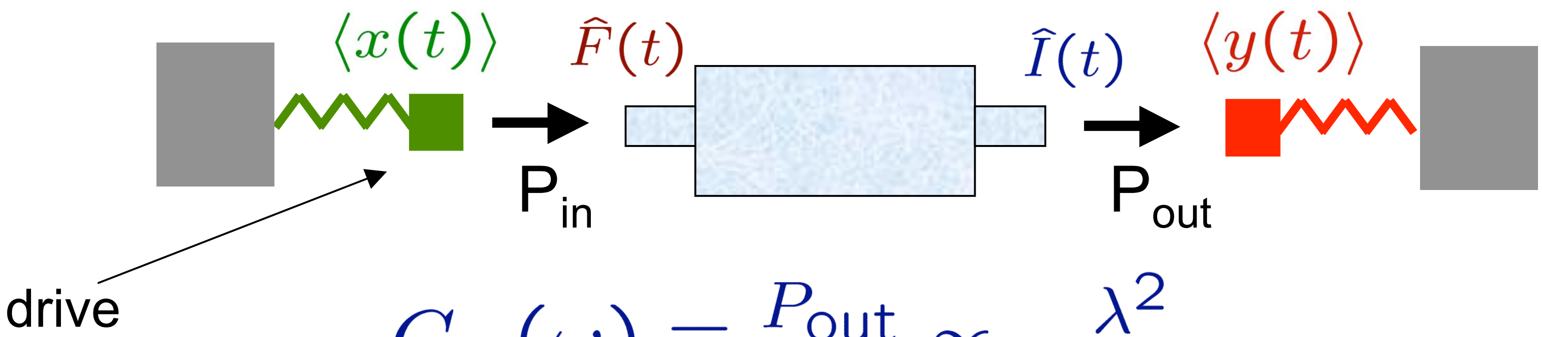
$$\bar{S}_I(\omega)\bar{S}_F(\omega) - [\text{Re } \bar{S}_{IF}(\omega)]^2 \geq \left(\frac{\hbar\lambda(\omega)}{2} \right)^2$$

- *A detector with "quantum ideal" noise?*
  - One where the product $S_I\ S_F$ reaches a minimum.
- *Reaching the quantum limit on the added requires a detector with "quantum ideal" noise….*

Power Gain

- Only have a quantum limit if our detector truly amplifies
- Need dimensionless measure of ***power gain….***

$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$



$$G_P(\omega) \equiv \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \propto \frac{\lambda^2}{\gamma_{\mathrm{in}}\gamma_{\mathrm{out}}}$$

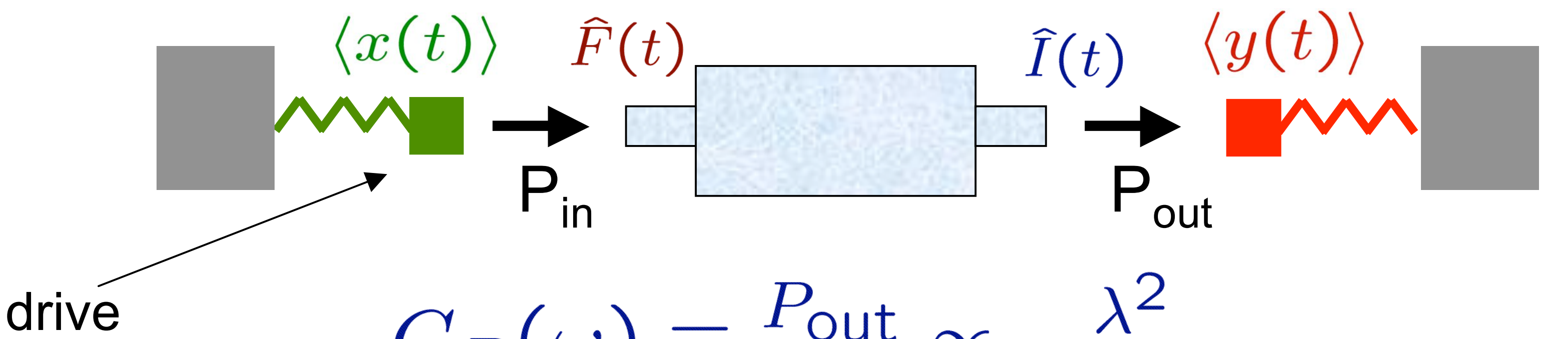
Need a large power gain!

Otherwise, we can't ignore the added noise of the next stage of amplification!

Power Gain

- Only expect a quantum limit if our detector truly amplifies
- Need to introduce the notion of a dimensionless ***power gain….***

$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$



$$G_P(\omega) \equiv \frac{P_{\text{out}}}{P_{\text{in}}} \propto \frac{\lambda^2}{\gamma_{\text{in}}\gamma_{\text{out}}}$$

- If detector has "ideal" quantum noise:

$$G_P(\omega) = \left(\frac{4k_B T_{eff}}{\hbar\omega} \right)^2$$

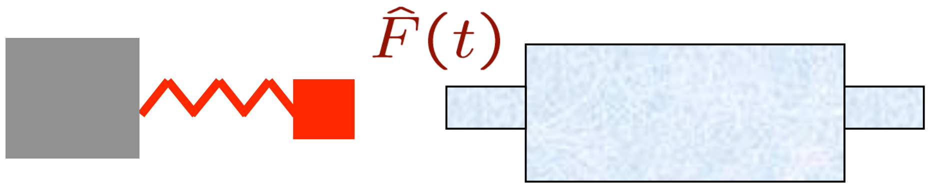
Power gain set by effective temperature!

- If also $G_P >> 1$:

S_{IF} must be real!

(correlations can't help beat QL!)

Minimum Added Noise



$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$

- Quantum noise constraint leads to the quantum limit…
- Consider a large power gain… cross-correlator S_{IF} is real

$$S_x(\omega) = \frac{\bar{S}_I}{A^2\lambda^2} + A^2|g(\omega)|^2\bar{S}_F - \frac{2\mathrm{Re}\ [g(\omega)]\bar{S}_{IF}}{\lambda}$$

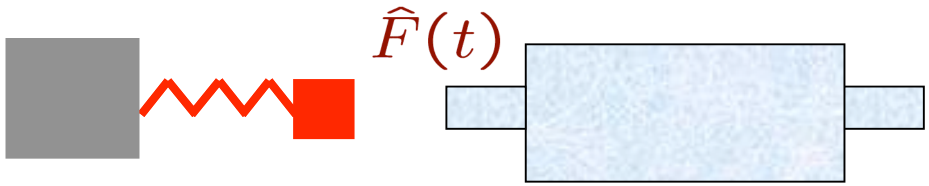
Intrinsic output noise of detector

Effect of back-action force noise

Three steps for reaching the quantum limit:

$$g(\omega) = \frac{1}{m}\frac{1}{\omega^2 - \Omega^2 + i\omega\gamma}$$

Minimum Added Noise



$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$

- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

$$S_x(\omega) = \frac{\bar{S}_I}{A^2 \lambda^2} + A^2 |g(\omega)|^2 \bar{S}_F - \frac{2\text{Re}\ [g(\omega)] \bar{S}_{IF}}{\lambda}$$

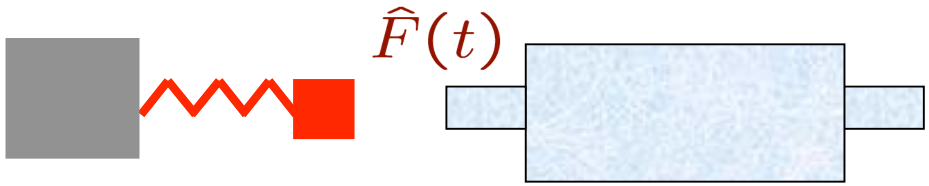
Intrinsic output noise of detector

Effect of back-action force noise

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling *A.*

Minimum Added Noise



$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$

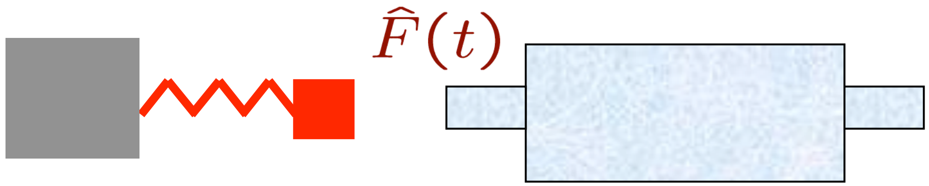
- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

$$S_x(\omega) = \frac{\bar{S}_I}{A^2\lambda^2} + A^2|g(\omega)|^2\bar{S}_F - \frac{2\text{Re }[g(\omega)]\bar{S}_{IF}}{\lambda}$$
$$\geq 2|g(\omega)|\left[\sqrt{\bar{S}_I\bar{S}_F/\lambda^2} - \frac{\cos\phi(\omega)\bar{S}_{IF}}{\lambda}\right]$$

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling *A.*

Minimum Added Noise



$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$

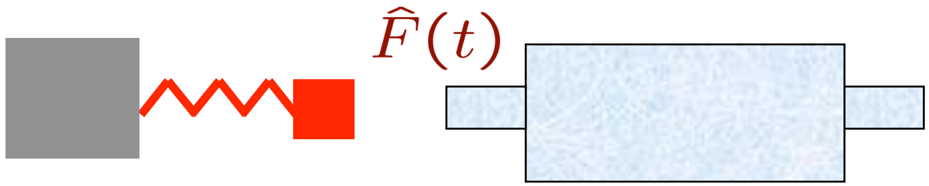
- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

$$\begin{aligned} S_x(\omega) &= \frac{\bar{S}_I}{A^2\lambda^2} + A^2|g(\omega)|^2\bar{S}_F - \frac{2\text{Re }[g(\omega)]\bar{S}_{IF}}{\lambda} \\ &\geq 2|g(\omega)|\left[\sqrt{\bar{S}_I\bar{S}_F/\lambda^2} - \frac{\cos\phi(\omega)\bar{S}_{IF}}{\lambda}\right] \end{aligned}$$

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling A
2. ***Use a quantum-limited detector!***

Minimum Added Noise



$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$

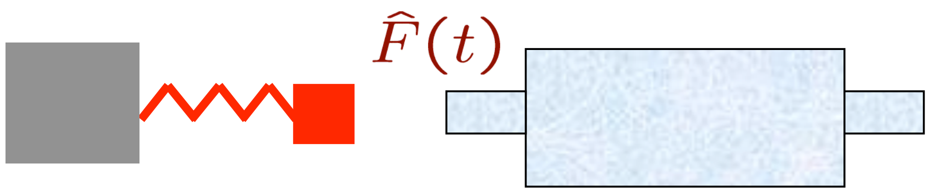
- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

$$\begin{aligned} S_x(\omega) &= \frac{\bar{S}_I}{A^2\lambda^2} + A^2|g(\omega)|^2 \bar{S}_F - \frac{2\mathrm{Re}\,[g(\omega)]\bar{S}_{IF}}{\lambda} \\ &\geq 2|g(\omega)| \left[\sqrt{\frac{\hbar^2}{4} + \frac{\bar{S}_{IF}^2}{\lambda^2}} - \frac{\cos\phi(\omega)\bar{S}_{IF}}{\lambda} \right] \end{aligned}$$

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling A
2. *Use a quantum-limited detector!*

Minimum Added Noise



- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

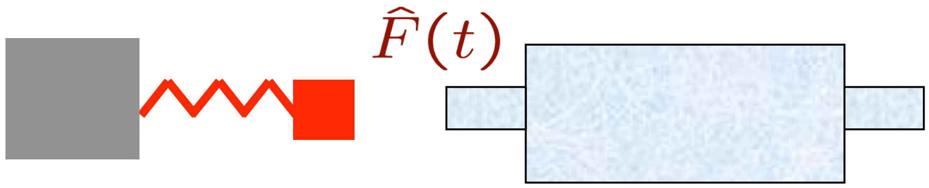
$$S_x(\omega) = \frac{\bar{S}_I}{A^2\lambda^2} + A^2|g(\omega)|^2 \bar{S}_F - \frac{2\mathrm{Re}\,[g(\omega)]\bar{S}_{IF}}{\lambda}$$

$$\geq 2|g(\omega)| \left[\sqrt{\frac{\hbar^2}{4} + \frac{\bar{S}_{IF}^2}{\lambda^2}} - \frac{\cos\phi(\omega)\bar{S}_{IF}}{\lambda} \right]$$

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling
2. *Use a quantum-limited detector!*
3. Tune the cross-correlator S_{IF}

Minimum Added Noise



$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$

- Quantum noise constraint leads to the quantum limit…
- Assume the limit of a large power gain ⇒ S_{IF} / λ is real

$$S_x(\omega) = \frac{\bar{S}_I}{\lambda^2 A^2} + A^2 |g(\omega)|^2 \bar{S}_F - \frac{2\text{Re } [g(\omega)] \bar{S}_{IF}}{\lambda}$$

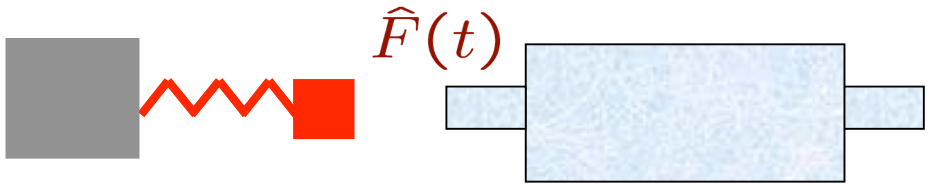
$$\geq \frac{\hbar \omega \gamma_{tot} / m}{(\omega^2 - \Omega^2)^2 + \omega^2 \gamma_{tot}^2} = S_{x,\text{zpt}}(\omega)$$

Same as zero point noise!

Three steps for reaching the quantum limit:

1. Balance back action and intrinsic noise via tuning coupling A
2. Use a quantum-limited detector!
3. Tune the cross-correlator S_{IF}

On resonance, ω = Ω



$$\langle \hat{I}(t) \rangle = A\lambda \langle x(t) \rangle$$

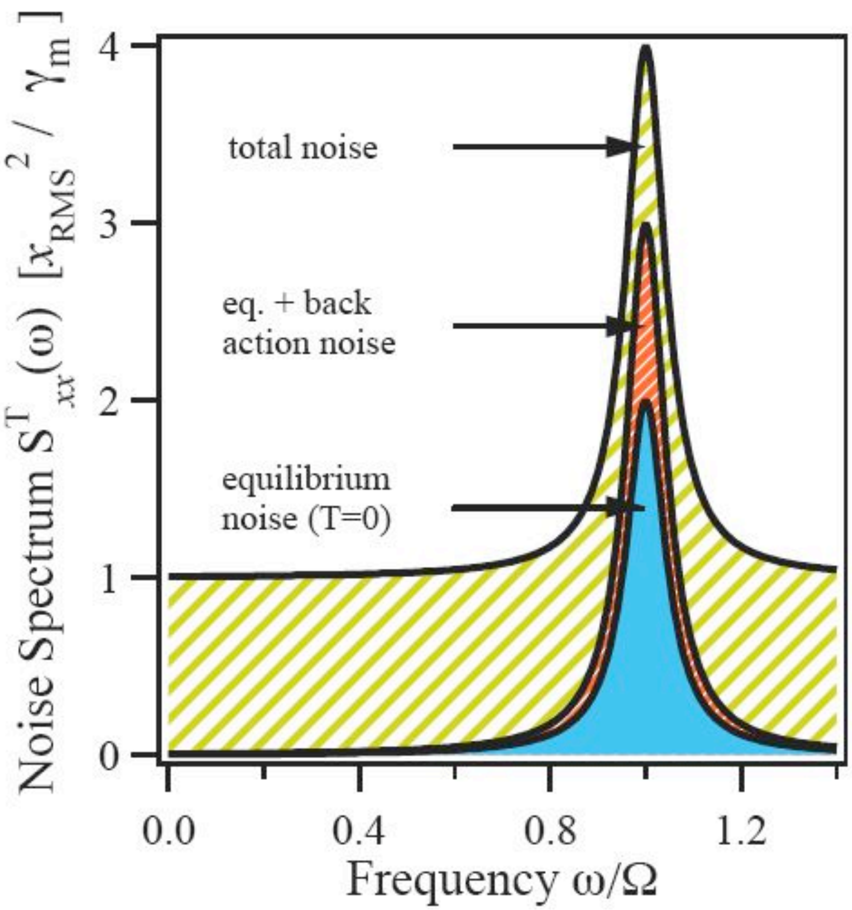
$$S_x(\omega = \Omega) \geq \frac{\hbar}{m\Omega} \cdot \frac{1}{\gamma_{tot}} = 2\,(\Delta x)^2 \frac{1}{\gamma_{tot}}$$

- The condition for an optimal coupling takes a simple form:

$$\frac{A_{opt}^2 \gamma}{\gamma_0 + A_{opt}^2 \gamma} = \frac{\hbar \Omega}{4 k_B T_{eff}}$$

- *At the quantum limit, the amplifier-oscillator coupling has to be weak enough to offset the large T_{eff} of the amplifier*

Detecting Zero Point Motion?



$$\hat{I}(t) = \lambda\langle x(t)\rangle + \xi(t)$$

- Pick coupling to minimize added noise *on resonance*
- *Oscillator peak is 4 times noise background…*

Experiments:

$$\frac{k_B T_N}{\hbar\omega/2} = \frac{\sqrt{S_I S_F}}{\hbar\lambda/2}$$

Naik, Schwab et al. 06: SET detector, 15×QL (*but…*)
using actual output noise? 525 × QL

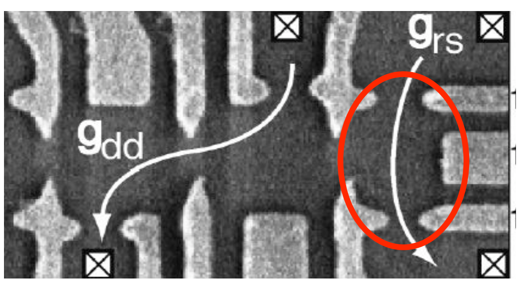
Flowers-Jacobs, Lehnert et al. 07: APC detector, (1700±400)*QL (measured!)

Meaning of "ideal noise"?

- *Key point: need to have a detector with "ideal" quantum noise to reach the quantum limit.*

$$S_I(\omega)S_F(\omega) - [\mathrm{Re}\ S_{IF}(\omega)]^2 \geq \hbar^2\,[\mathrm{Re}\ \lambda(\omega)]^2$$

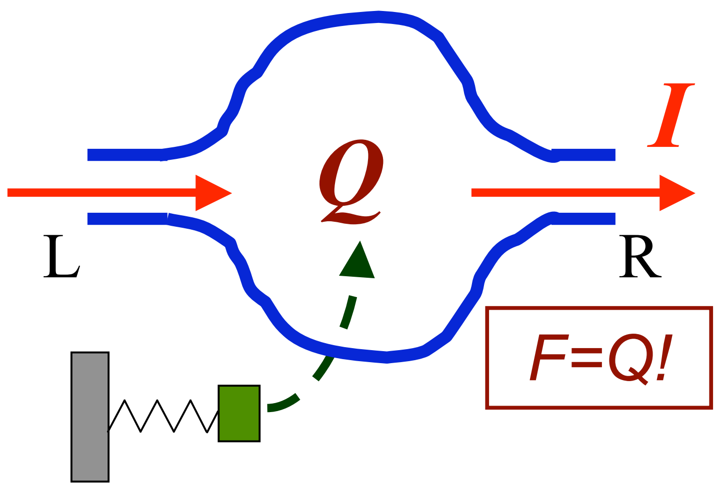
- *What does this mean?*
  - Detector ***cannot*** be in a thermal equilibrium state;
  - More concrete: "no wasted information"
    e.g. generalized QPC detector
    - scattering matrix must satisfy constraints related to "wasted information" (Pilgram & Buttiker; A.C., Stone & Girvin)



$$\frac{d}{d\varepsilon}(\beta - \varphi) = 0$$
$$\frac{\frac{dT}{d\varepsilon}}{T(1-T)} = \frac{1}{\mathcal{C}}$$

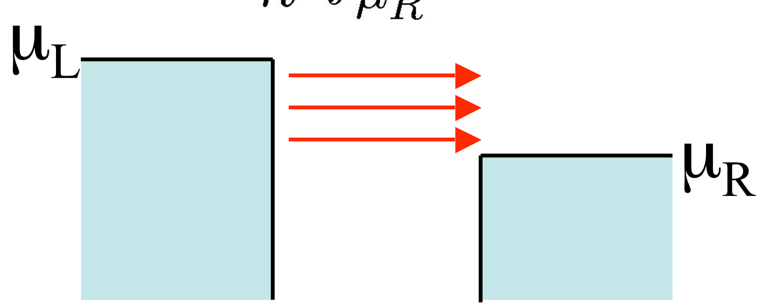
Mesoscopic Scattering Detector

(AC, Girvin & Stone 03)



$$s(\varepsilon) = \begin{pmatrix} \sqrt{1-T}e^{i\beta} & \sqrt{T}e^{i\varphi'} \\ \sqrt{T}e^{i\varphi} & -\sqrt{1-T}e^{i\beta'} \end{pmatrix}$$

$$I_0 = \frac{e^2}{h}\int_{\mu_R}^{\mu_L} d\varepsilon T(\varepsilon)$$



T depends on oscillator:

$$T(\varepsilon) = T_0(\varepsilon) + \frac{dT(\varepsilon)}{dx}\cdot x$$

$$\langle I \rangle = I_0 + A\lambda\langle x(t)\rangle$$

Insisting on "ideal" noise puts constraints on s-matrix:

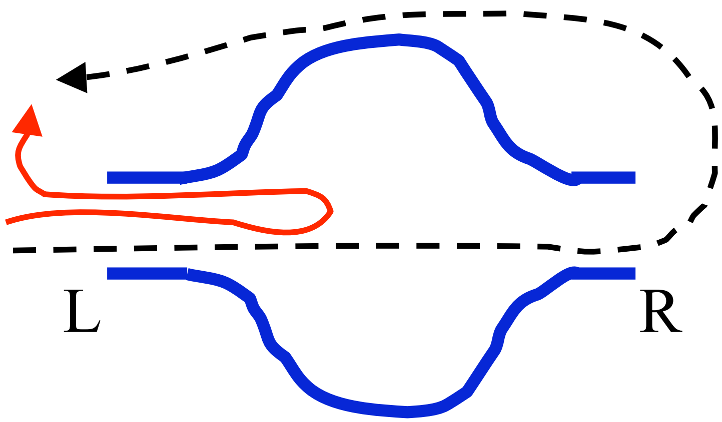
$$\begin{aligned} \frac{d}{dx}(\beta - \varphi) &= 0 \\ \left[\frac{\frac{dT}{dx}}{T(1-T)}\right](\varepsilon) &= \frac{1}{\mathcal{C}} \end{aligned}$$

Wasted Information?

(AC, Girvin & Stone 03)

Phase Info:

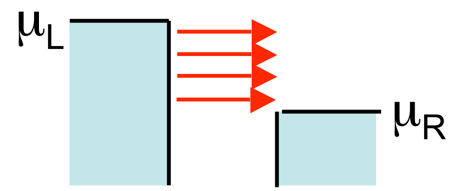
- try to learn more by doing an interference expt.



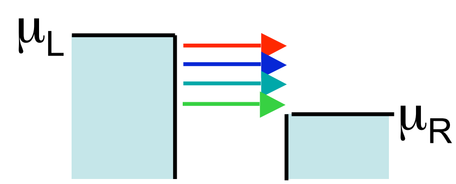
$$\frac{d}{dx}(\beta - \varphi) = 0$$

Info in $T(\varepsilon)$:

- try to learn more by using the energy-dependence of T

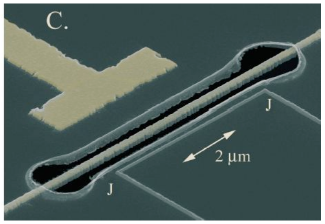


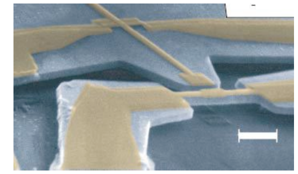
versus



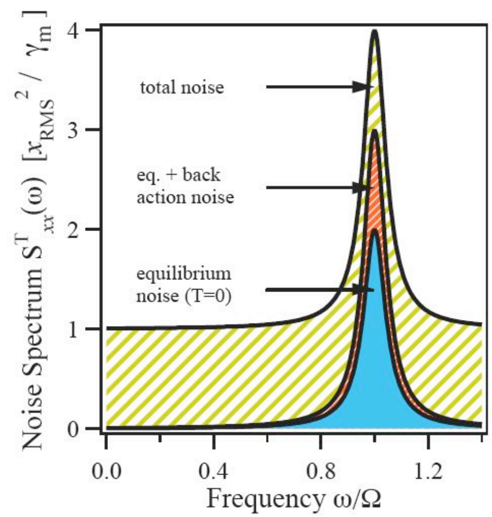
$$\left[\frac{\frac{dT}{dx}}{T(1-T)}\right](\varepsilon) = \frac{1}{\mathcal{C}}$$

Conclusions





- T_{eff} of a non-equilibrium system:
  - Defined by the detector's quantum noise spectrum
  - Characterizes asymmetry between absorption and emission of energy
  - In general, is frequency dependent
- Quantum Limit
  - There are quantum constraints on noise
  - Reaching the quantum limit requires a detector with "ideal" noise



*Clerk, Phys. Rev. B **70,** 245306 (2004)*
*Clerk & Bennett, New. J. Phys. **7**, 238 (2005)*
Clerk, Girvin, Marquardt, Devoret & Schoelkopf, RMP (soon!)